CS323 - Winter 97 - Homework #4

Due date: Thursday, March 13th

February 25, 1997

This assignment is much easier than it looks (the first problem)[1], and much harder than it looks (the second one).

1 Three wise men in PVS

For the following problem you must have had read the paper [Hal95] which summarizes properties of formalizing knowledge, and especially pages 1–7 of it (you were required to read it a few weeks ago).

Theorem 1.1 (copied from [FHMV95] p. 95) *First order logic is expressive enough to capture the propositional modal logic S5 (without common knowledge). Put more precisely, we build a translation from the modal logic of knowledge S5 to first order logic.*

Let Φ be a set of primitive propositions, let the vocabulary $\Phi*$ consist of a unary predicate P corressponding to each primitive proposition p in Φ, as well as binary predicates $R_1, ..., R_n$ for each agent[2]. We now define a translation from formulas in $\mathcal{L}_n(\Phi)$ to first-order formulas over $\Phi*$, so that for every formula $\varphi \in \mathcal{L}_n(\Phi)$ there is a corresponding first-order formula $\varphi*$ with one free variable x:

- $p* = P(x)$
- $(\neg\varphi)* = \neg(\varphi*)$
- $(\varphi \wedge \psi)* = \varphi * \wedge \psi*$
- $(K_i\varphi)* = \forall y(R_i(x, y) \implies \varphi * (y))$*, where y is a new variable not appearing in $\varphi*$ and $\varphi * (y)$ is the result of replacing all occurences of x in $\varphi*$ by y.*

[1] In fact, this assignment is much easier than the MCP you dealt with in the previous HW assignment

[2] these R_is will replace the knowledge modalities K_i for the different agents, in the translation that we hereforth build.

Next we provide a mapping from a Kripke structure $M = (S, \pi, K_1, ..., K_n) \in \mathcal{M}_n(\Phi)$[3] *to a relational $\Phi*$-structure $M*$. The domain of $M*$ is S. For each primitive proposition $p \in \Phi$, we let $P^{M*} = \{s \in S : \pi(s)(p) = \mathbf{true}\}$, and let $R_i^{M*} = K_i$.*

Then

1. *$(M, s) \models \varphi$ iff $(M*, V) \models \varphi * (x)$, where $V(x) = s$. Intuitively, this says that $\varphi*$ is true of exactly the domain elements corresponding to worlds s for which $(M, s) \models \varphi$.*

2. *φ is valid with respect to $\mathcal{M}_n(\Phi)$ iff $\forall x \varphi * (x)$ is a valid first-order formula.*

Hal95 esp. pages 1-7.

Problem 1.2 Formalize and solve the 3-wise-men problem (discussed in class) using the PVS. *You are required to solve only the "easy" direction (i.e., that on the third question, they answer "yes"). Hint: You will find the above Theorem useful... and of course you can use any reference material you want (including some developed formalizations for other theorem provers).*

2 Trivia

Problem 2.1 *In the following question you are asked to give definition and description of the givven terms. The deeper and more precise answers will be rewarded with better scores (Hint: I am looking for more than a line here).*

1. *What is Elaboration Tolerance*

2. *What is Common Sense Reasoning.*

References

[FHMV95] R. Fagin, J.Y. Halpern, Y. Moses, and M.Y. Vardi. *Reasoning about knowledge.* MIT Press, 1995.

[Hal95] Joseph Y. Halpern. Reasoning about Knowledge. In J.A.Robinson D.M. Gabbay, C.J.Hogger, editor, *Handbook of Logic in Artificial Intelligence and Logic Programming, Vol.4.* Oxford University Press, 1995.

[3] S is the set of worlds, π is the truth value assignment for each of the worlds $s \in S$, and the propositions $p \in \Phi$, and the K_is are the knowledge modalities (other words - the accessibility relations).