CS323: Circumscription

- Two readings for today:
  - *Circumscription--A Form of Non-Monotonic* _________. [McC80]

    ____
    _______________________. [McC86]

    ______________. [Lif93]

Circumscription -- A Form of Nonmonotonic Reasoning: Motivation

- Qualification problem: no way to specify exact conditions for performance of an action
- Solution to this is circumscription:
  - rule of conjecture used to jump to conclusions
  - assumes propositions false unless said otherwise (CWA assumption for databases)
  - the objects that are shown to have property P are the only ones.

Circumscription -- A Form of Nonmonotonic Reasoning: Examples

- There are 3 blocks: A, B, and C.

  (From that you assume that there are no other blocks) Domain circumscription
- Onn, Stefan, and Lauren are all getting As in CS323. Predicate circumscription
- Boats (can use to cross a river unless something prevents you), tools.

Circumscription -- A Form of Nonmonotonic Reasoning: What's Monotonicity?

Monotonicity is a feature of most logical systems. Say A and B are sets of sentences, and q is a proposition:

$$\text{if } A \mid\!- q \text{ and } A \subset B\text{, then } B \mid\!- q$$

(adding more sentences to the premises only increases the number of conclusions!)

Circumscription -- A Form of Nonmonotonic Reasoning: What's Nonmonotonicity?

- So then nonmonotonicity is when:
  A |- q and A $\subset$ B, but not necessarily B |- q
- For example,
  A = {broken(boat), ..}
  q = $\neg$cross(person)
  B = A $\cup${spans(river, bridge)}
- Also applies to semantic notion (|=)

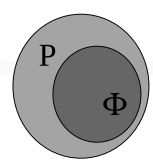
Circumscription -- A Form of Nonmonotonic Reasoning

- Seems to be a rule of conjecture in life, but in puzzles a rule of inference
- Against probability/fuzzy logic:
  - probability of MCP problem not meaningful,
  - probability of bridge given that even less meaningful
  - people consider normal case first -- not the sample space of all possibilities

Circumscription -- A Form of Nonmonotonic Reasoning: Formalism

- Predicate Circumscription: minimize extent of predicate P
- Given a theory A, two ways to formalize:
  1. Semantic: Pick out the models of A which minimize the extent of some domain or predicate
  2. Syntactic: Append a sentence to A to do minimization syntactically.

Circumscription -- A Form of NMR: Predicate Circumscription



- Some notation first:
  - We abbreviate P(x) for P(x_1, …, x_n)
  - We define $\Phi \leq P$ as $\forall x.\ \Phi(x) \rightarrow P(x)$,
  - We also say $\Phi = P$ as $\forall x.\ \Phi(x) \equiv P(x)$
  - A(Φ) is theory A, with all occurrences of P replaced with Φ
- Magical circumscription of P in A(P) = Circ[A(P); P]:

  $A(P) \wedge \forall \Phi.[(A(\Phi) \wedge \ \Phi \leq P) \rightarrow \Phi = P\ ]$

Circumscription -- A Form of NMR: Predicate Circumscription

- Again:

  Circ[A(P); P] =

  $A(P) \wedge \forall\Phi.[(A(\Phi) \wedge \Phi \leq P) \rightarrow \Phi = P]$

- Alternate, equivalent form:

  Circ[A(P); P] = $A(P) \wedge \neg\exists\Phi.(A(\Phi) \wedge \Phi < P)$

- McCarthy in McC80 uses schemas rather than second-order logic. In McC86 back to second-order.

Circumscription -- A Form of NMR: Predicate Circumscription

- Can also do joint circumscription over two predicates P and Q. Circ[A(P, Q); P, Q] =

  $A(P, Q) \wedge$

  $\forall\Phi, \Psi.[(A(\Phi, \Psi) \wedge \Phi \leq P \wedge \Psi \leq Q) \rightarrow$

  $(\Phi = P \wedge \Psi = Q)]$

- Can also allow other symbols (predicates, constants, functions) to vary (more on this later).

Circumscription -- A Form of NMR: Examples of Predicate Circumscription

Simpler examples from Lif93:

1. A(P) is P(a):
2. A(P) is $\neg$P(a):
3. A(P) is P(a) $\wedge$ P(b):
4. A(P) is P(a) $\vee$ P(b):
5. A(P) is P(a) $\rightarrow$ P(b):
6. A(P) is $\forall$x.Px:
7. A(P) is $\forall$x.Qx$\rightarrow$ Px:

Circumscription -- A Form of Nonmonotonic Reasoning: Extensions

- Irrelevant symbols in circumscription get factored out:

  Say $A(P) = B \wedge C(P)$.

  $Circ[A(P); P]$

  $= A(P) \wedge \forall\Phi.[(A(\Phi) \wedge \ \Phi \leq P) \rightarrow \Phi = P\ ]$

  $= B \wedge C(P) \wedge \forall\Phi.[(\cancel{B} \wedge\ C(\Phi) \wedge\ \Phi \leq P) \rightarrow \Phi = P\ ]$

  $= B \wedge Circ[C(P); P]$

- Domain circumscription:

  $A \wedge \forall\Phi.[Axiom(\Phi)\ \wedge\ A^{\Phi} \rightarrow \forall x.\Phi(x)]$

Circumscription -- A Form of NMR: Semantic Way of Circumscription

Model theory of circumscription:

- order models of A by relation $\leq_P$, where two models
  M(A) $\leq_P$ N(A) iff
  (extension of P in M) $\subseteq$ (extension of P in N) and
  everything else same
- pick the $\leq_P$ - minimal models of A. (Those models M(A) such that $\neg\exists$M'(A). M'(A) $<_P$ M(A))
- Circ[A(P); P] is satisfied in any of these minimal models.

Circumscription -- A Form of NMR: Varying Symbols

- From Lif93: (MCC86 refers to this in the bird example, so I should explain here.)
- Certain symbols can "vary" in the circumscription. We denote this as Circ[A(P, Z); P; Z], where Z is the relation/constant/function symbol to be varied.
- There are parallel semantic and syntactic ways of describing variation:

Circumscription -- A Form of NMR: Varying Symbols

- For Circ[A(P, Z); P; Z], we redefine our model ordering relation to be:
  - $M(A) \leq_P N(A)$ iff
    (extension of P in M) $\subseteq$ (extension of P in N) and
    everything else same **except Z**
- This means that we don't have to worry about the Zs being the same in M and N, which means that they are allowed to vary.

Circumscription -- A Form of NMR: Varying Symbols

- Syntactically, we write the circumscription formula as:

  Circ[A(P, Z); P; Z] =

  $$A(P, Z) \wedge \forall \Phi, \varsigma.[(A(\Phi, \varsigma) \wedge \Phi \leq P) \rightarrow \Phi = P]$$
- We can set ς to be whatever we like, in order to help satisfy the LHS of the implication.
- Remember, Z can be a constant, function, or predicate symbol!

Circumscription -- A Form of NMR: Conclusions

- Circumscription is not a nonmonotonic logic, but a type of reasoning augmenting FOL
- More expressive than default logic
- Both McCarthy and Lifschitz propose using circumscription in a reasoning program, where it is described what predicates are circumscribed how.
- (obvious) circumscription may lead to different results, depending on how it is formalized.

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Applications of Circumscription to Formalizing Common Sense Knowledge

- Introduces formula circumscription, where instead of minimizing the extent of a predicate, you minimize the extent of a formula:

$A(P) \wedge \forall\Phi.[(A(\Phi) \wedge E(\Phi) \leq E(P)) \rightarrow E(\Phi) = E(P)]$

- Instead of minimizing P directly (predicate circumscription), you minimize E(P).
- More expressive than just trying to minimize the extension of predicates

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Applications of Circumscription: Seven Uses of an Elephant

1. Communication convention: If not mentioned, assume not/default
2. Database convention: only certain predicates CWAed
3. Rule of conjecture: “Most birds fly.”
4. Policy representation: “The meeting is on Wed. unless there is another decision.”
5. Streamlined version of probabilities
6. Auto-epistemic reasoning
7. Common sense physics and psychology

Applications of Circumscription: Abnormalities

- Filter in all the qualifications into one predicate, *ab z*, where *z* represents some aspect of an object.
- *Ab z* represents all the conditions that could make some property of an object not hold:

$$\forall x. \neg ab(aspect1(x)) \rightarrow \neg flies(x)$$

“Most objects do not fly.”

- Then, minimize *ab.*

Applications of Circumscription

- Inheritance hierarchies
- UNH
- Nixon Diamond, Vancouver vs. Toronto
- General Is-a hierarchy treatment
- Blocksworld (changing color and location)
- More birds flying

Applications of Circumscription: Prioritized Circumscription

- Consider the ordering

  $ab \leq ab' \equiv \forall x.\ ab(x) \rightarrow ab'(x)$
- For each $aspect_i$, we can define an ordering

  $ab \leq_i ab' \equiv \forall x.\ ab(aspect_i(x)) \rightarrow ab'(aspect_i(x))$
- Then we can have an ordering on the orderings, say on 1 and 2:

  $ab \leq_{1<2} ab' \equiv ab \leq_2 ab' \wedge [ab =_2 ab' \rightarrow ab \leq_1 ab']$
- This means to see if $ab \leq_{1<2} ab'$, check if the abs are ordered wrt to $aspect_{2.}$ If they are equal wrt to 2, then fall back on $aspect_1$.

Applications of Circumscription: Considerations and Remarks

- Certain forms of Circ[A(P); P] are collapsible to FOL.
- Circumscription can be viewed as a process of compiling higher-order logic into FOL
- Circumscription is more computable than Reiter's default logic
- Still need to add hints to the reasoning program as to how to circumscribe
- Still other undiscovered ways to use nonmonotonic reasoning