Around merging strategy-proofness

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Problem Description

Belief merging : aim at defining the beliefs/goals of a group of agents from the beliefs/goals of each members of the group.
In that work, we try to find criteria to study and compare merging operators (see the following four criteria) :

- rationality (i.e., logical properties)
- complexity (computational efficiency)
- strategy-proofness
- discrimination power (it is natural to prefer operators leading to consistant merged based as strong as possible).

Among them, we are particularly interested in strategy-proofness of merging operators, because this is a new criterion for that operators.
As no usual merging operator fully satisfies the four criteria, we define new merging operators' families, quota operators and $\triangle^{\textsc{Gmin}}$ operators, which appear as interesting trade-offs.

Problem Desciption

- Strategy-proofness
  - General issue of strategy-proofness : can an agent rig the merging process by lying on her real beliefs/goals ?
  - The strategy-proofness of a merging operator is a suitable property in order to keep the fairness between agents.
  - First aim of our work : find the boundary between stategy-proofness and manipulability for the merging operators from the literature.
- New merging operators
  - Quota operators : any model of the merged base satisfies "sufficiently many" (that is more than the quota value) bases.
  - G_{min} operators : define from a distance, and refine the quota operators.

Problem Description : Beliefs/goals merging

Beliefs and goals are distinct notions, but belief merging operators can be used either for merging belief bases or goal bases. In the following, we talk of belief bases to denote belief or goal bases.

- Belief base : consistent formula representing the beliefs/goals of an agent, denoted by K
- Profile : multi-set of belief bases representing the beliefs of a group of agents, denoted by $E = \{K_1, K_2, ..., K_n\}$
- Integrity constraint : consistent formula the merged base has to satisfy (for example physical constraints, laws, etc...) denoted by μ
- Merging result (view as a belief base) : denoted by $\Delta_\mu(E)$

Problem Description : Example

- ▷ Marie does not want to go to the restaurant : $\phi_1 = \{\neg r\}$.
- ▷ Alain does not want to stay at home (i.e. he wants to go to the restaurant, or to the movie) $\phi_2 = \{r \vee m\}$.
- ▷ Pierre wants to go to a restaurant for diner, but not to the movie $\phi_3 = \{r \wedge \neg m\}$.

By using a majority merging operator (the operator $\Delta^{d_H,\Sigma}$) they have to go to the restaurant for diner, and not to the movie.

But, if Marie lies and claims that :

- ▷ Marie wants to go to the movie and not to the restaurant $\phi_1' = \{\neg r \wedge c\}$

then the goal of the group will be to go to the movie or to the restaurant.

Marie still may avoid to go out for diner !

Problem Description : Model-based merging operators

Given a profile E and an integrity constraint μ, the models of the merged base are the models of μ which are minimal for a total pre-order mapped to E, denoted by $\leq_E$: $[\triangle_\mu(E)] = \min([\mu], \leq_E)$.
To define $\leq_E$, we usually use :

- d : a (pseudo-)distance between interpretations, that is a total function d s.t. $d(\omega, \omega') = d(\omega', \omega)$ and $d(\omega, \omega') = 0$ iff $\omega = \omega'$. Example : Hamming distance d_H, Drastic distance d_D ($d_D(\omega, \omega') = 0$ if $\omega = \omega'$ and 1 otherwise).
- f : an aggregation function f associates a nonnegative real number to every finite tuple of nonnegative real numbers and satisfies some natural properties (non-decreasingness,etc). For example, the sum Σ or the leximax G_{max} are often used.

And then :

$$d(\omega, K) = min_{\omega' \models K} d(\omega, \omega') \qquad d_f(\omega, E) = f_{K_i \in E}(d(\omega, K_i))$$

$$\omega \leq_E^{d,f} \omega' \text{ iff } d_f(\omega, E) \leq d_f(\omega', E) \qquad [\triangle_\mu^{d,f}(E)] = \min([\mu], \leq_E^{d,f}).$$

Problem Description : Example, merging with $\Delta^{d_H,\Sigma}$

We consider the two variables m(movie) and r(restaurant).
$[K_1] = \{00, 10\}, [K_2] = \{01, 10, 11\}$, $[K_3] = \{01\}$ and $\mu = \top$.

ω	$d_H(\omega, K_1)$	$d_H(\omega, K_2)$	$d_H(\omega, K_3)$	$\Delta_{\mu}^{d_H,\Sigma}(\{K_1, K_2, K_3\})$
00	0	1	1	2
01	**1**	**0**	**0**	**1**
10	0	0	2	2
11	1	0	1	2

$\Delta_{\mu}^{d_H,\Sigma}(\{K_1, K_2, K_3\}) = \{01\}$

Problem Description : Syntax-based merging operator

We consider a belief base K and an integrity constraint μ, and define maximal consistant subsets of formula of K with respect to μ, $\textsc{maxcons}(K, \mu)$ is the set of all M that satisfy : $M \subseteq K \cup \{\mu\}$, and $\mu \in M$, and If $M \subset M' \subseteq K \cup \{\mu\}$, then M' is not consistent.
When maximality must be taken w.r.t. cardinality, we use the notation $\textsc{maxcons}_{card}(K, \mu)$.
Then each operator select some maximal consistant subsets with different criteria :

- $\Delta_\mu^{C1}(E) = \bigvee\{M \in \textsc{maxcons}(\bigcup K_i, \mu)\}$.
- $\Delta_\mu^{C3}(E) = \bigvee\{M \mid M \in \textsc{maxcons}(\bigcup K_i, \top)$ and $M \cup \{\mu\}$ consistent$\}$.
- $\Delta_\mu^{C4}(E) = \bigvee\{M \in \textsc{maxcons}_{card}(\bigcup K_i, \mu)\}$.
- $\Delta_\mu^{C5}(E) = \bigvee\{M \cup \{\mu\} \mid M \in \textsc{maxcons}(\bigcup K_i, \top)$ and $M \cup \{\mu\}$ consistent $\}$ if non empty and μ otherwise.

Problem Description : Example, merging with $\triangle^C$

- $E = \{K_1, K_2, K_3\}$ with
  - $K_1 = \{\neg r\}$ (Marie's choice)
  - $K_2 = \{m \vee r\}$ (Alain's choice)
  - $K_3 = \{\neg m \wedge r\}$ (Pierre's choice)
- MAXCONS$(E, \top) = \{\{\neg r, c \vee r, \top\}, \{c \vee r, \neg c \wedge r, \top\}\}$.
- $\Delta_\mu^{C1}(E) = \Delta_\mu^{C3}(E) = \Delta_\mu^{C4}(E) = \Delta_\mu^{C5}(E) \equiv (m \wedge \neg r) \vee (\neg m \wedge r)$.

Problem Description : Strategy-proofness

- i a satisfaction index : $\mathcal{L} \times \mathcal{L} \longrightarrow \mathbb{R}$
  $i(K, K_\Delta)$ *gives a value to the satisfaction of an agent whose belief base is* K *relating to a merged base* K_Δ*.*
- Δ is *strategy-proof* for i iff there is no integrity constraint μ, profile E, base K and base K' such that
  $i(K, \Delta_\mu(E \sqcup \{K'\})) > i(K, \Delta_\mu(E \sqcup \{K\}))$.
  The satisfaction of the agent whose belief base is K *is maximal by giving her true belief base.*
- A profile E is said to be *manipulable* by a base K for index i given a merging operator Δ and an integrity constraint μ iff there exists a base K' s.t. $i(K, \Delta_\mu(E \sqcup \{K'\})) > i(K, \Delta_\mu(E \sqcup \{K\}))$.
- Restricted stategies :
  - dilatation strategy-proofness : $K \models K'$
  - erosion strategy-proofness : $K' \models K$

Problem Description : Satisfaction indexes

- Weak drastic index : $i_{d_w} = 1$ if $K \wedge K_\Delta$ is consistent, $i_{d_w} = 0$ otherwise.
  For that index, an agent is either fully satisfied if the merged base is consistant with her beliefs, or not at all if not.
- Strong drastic index : $i_{d_s} = 1$ if $K_\Delta \models K$, $i_{d_s} = 0$ otherwise.
  For that index, an agent is either fully satisfied if the merged base is stronger than her beliefs, or not at all if not.
- Probabilistic index i_p : $i_p(K, K_\Delta) = \frac{\#([K] \cap [K_\Delta])}{\#([K_\Delta])}$.
  That index is not drastic, leading to a more gradual satisfaction notion : the more compatible the merged base with the agent's base the more satisfied the agent.

Strategy-proofness for $i_p \Rightarrow$ Strategy-proofness for i_{d_w} and i_{d_s}

Problem Description : Example, manipulability of Δ^{d_H}

We had $[\Delta_\mu^\Sigma(\{K_1, K_2, K_3\})] = \{01\}$, so $i_{d_w}(K_1, \Delta_\mu^{d_H,\Sigma}(\{K_1, K_2, K_3\})) = 0$.
With $[K_1'] = \{10\}$ we get $[\Delta_\mu^\Sigma(\{K_1', K_2, K_3\})] = \{01, 10, 11\}$ and
$i_{d_w}(K_1, \Delta_\mu^{d_H,\Sigma}(\{K_1', K_2, K_3\})) = 1$.

ω	K_1	K_1'	K_2	K_3	$\Delta_\mu^{d_H,\Sigma}(\{K_1,K_2,K_3\})$	$\Delta_\mu^{d_H,\Sigma}(\{K_1',K_2,K_3\})$
00	0	1	1	1	2	3
01	**1**	**2**	**0**	**0**	**1**	**2**
10	0	**0**	**0**	**2**	2	**2**
11	1	**1**	**0**	**1**	2	**2**

Results : Strategy-proofness

Numerous parameters have an effect on the merging operators strategy-proofness : a lot of them are not strategy-proof in general, but become strategy-proof when considering some restrictions, as :

- Integrity constraint : some operators are not strategy-proof when there is an integrity constraint, while they are when there is not (or when the constraint is true).
- Initial base : imposing the initial belief base to be complete (that is with one model ω, denoted by K_ω) may avoid having some strategies.
- Number of bases involved in the merging process : the critical case is when there are only two bases.

For the model-based operators, the distance or the pseudo-distance and the aggregation function used may also change the strategy-proofness of these operators.

Results : Strategy-proofness

We have get some very general results for model-based merging operators :

- For any pseudo-distance d' and aggregation function f :
  - *dilatation strategy-proofness for the three indexes*
  - *for the drastic indexes, strategy-proof for K' complete $\Rightarrow$ strategy-proof.*
- For the drastic distance d_D and any aggregation function f :
  - *strategy-proofness*
- For any distance, and the aggregation function Σ :
  - *for the drastic indexes, erosion strategy-proof for K' complete $\Rightarrow$ strategy-proof*
  - *strategy-proof for the three indexes if the initial belief base is complete*
  - *stategy-proofness for i_{d_w} or i_{d_s} for two agents without constraint*

Other results are given in the following tables.

Results : Strategy-proofness for i_p

$\mathbf{sp}$ means "strategy-proof" and $\overline{sp}$ means "not strategy-proof".

$\#(E)$	K	μ	$\Delta^{d_H,\Sigma}$	$\Delta^{d_H,G_{max}}$	Δ^{C1}	Δ^{C3}	Δ^{C4}	Δ^{C5}
2	K_ω	$\top$	$\mathbf{sp}$	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$
		μ	$\mathbf{sp}$	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$
	K	$\top$	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$
		μ	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$
> 2	K_ω	$\top$	$\mathbf{sp}$	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$
		μ	$\mathbf{sp}$	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$
	K	$\top$	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$
		μ	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$

For the probabilistic index i_p, there is not strategy-proof except when the initial belief base is complete for $\Delta^{d_H,\Sigma}$.

Results : Strategy-proofness for i_{d_w}

$\#(E)$	K	μ	$\Delta^{d_H,\Sigma}$	$\Delta^{d_H,G_{max}}$	Δ^{C1}	Δ^{C3}	Δ^{C4}	Δ^{C5}
2	K_ω	$\top$	**sp**	$\overline{sp}$	**sp**	**sp**	$\overline{sp}$	**sp**
		μ	**sp**	$\overline{sp}$	**sp**	$\overline{sp}$	$\overline{sp}$	**sp**
	K	$\top$	**sp**	$\overline{sp}$	**sp**	**sp**	$\overline{sp}$	**sp**
		μ	$\overline{sp}$	$\overline{sp}$	**sp**	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$
> 2	K_ω	$\top$	**sp**	$\overline{sp}$	**sp**	**sp**	$\overline{sp}$	**sp**
		μ	**sp**	$\overline{sp}$	**sp**	$\overline{sp}$	$\overline{sp}$	**sp**
	K	$\top$	$\overline{sp}$	$\overline{sp}$	**sp**	**sp**	$\overline{sp}$	**sp**
		μ	$\overline{sp}$	$\overline{sp}$	**sp**	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$

For the weak drastic index i_{d_w}, only $\Delta^{d_H,G_{max}}$ and Δ^{C4} are manipulable. The other operators are strategy-proof in some cases, and Δ^{C1} is always strategy-proof for that index.

Results : Strategy-proofness for i_{d_s}

$\#(E)$	K	μ	$\Delta^{d_H,\Sigma}$	$\Delta^{d_H,G_{max}}$	Δ^{C1}	Δ^{C3}	Δ^{C4}	Δ^{C5}
$=2$	K_ω	$\top$	**sp**	**sp**	**sp**	**sp**	$\overline{sp}$	**sp**
		μ	**sp**	$\overline{sp}$	**sp**	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$
	K	$\top$	**sp**	$\overline{sp}$	**sp**	**sp**	$\overline{sp}$	**sp**
		μ	$\overline{sp}$	$\overline{sp}$	**sp**	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$
>2	K_ω	$\top$	**sp**	$\overline{sp}$	**sp**	**sp**	$\overline{sp}$	**sp**
		μ	**sp**	$\overline{sp}$	**sp**	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$
	K	$\top$	$\overline{sp}$	$\overline{sp}$	**sp**	**sp**	$\overline{sp}$	**sp**
		μ	$\overline{sp}$	$\overline{sp}$	**sp**	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$

For the strong drastic index i_{d_s}, only Δ^{C4} is manipulable. The other operators are strategy-proof in some cases, and Δ^{C1} is always strategy-proof for that index.

Results : Quota operators

Let k be an integer, $E = \{K_1, \ldots, K_n\}$ be a profile, and μ be a formula. The k*-quota merging operator*, denoted $\triangle^k$, is defined in a model-theoretic way as :

- $[\triangle^k_\mu(E)] = \{\omega \in [\mu] \mid \forall K_i \in E \; \omega \models K_i\}$ if non empty,
- $\{\omega \in [\mu] \mid \#(\{K_i \in E \mid \omega \models K_i\}) \geq k\}$ otherwise.

Essentially, this definition states that the models of the result of the k-quota merging of profile E under constraints μ are the models of μ which satisfy at least k bases of E. When there is no conflict for the merging, i.e., $\bigwedge E \wedge \mu$ is consistent, the result of the merging is simply the conjunction of the bases.

Results : Quota operators

- rationality : quite good, as quota operators satisfy all the logical properties except two.
- complexity : it is in a low level of the boolean hierarchy, since the problem of knowing if a formula is a consequence of the merged base is *BH(2)-complete*.
- strategy-proofness : quota operators are strategy-proof for the three indexes.
- discrimination power : low, compared to other merging operators.

Even if they are not fully rational and discriminating, quota operators exhibit "low complexity" and are strategy-proof.

Results : G_{min} operators

It is possible to constrain further the quota operators so as to get operators with a higher discriminating power, i.e., allowing more inferences to be drawn. We provide here a full family of such operators. As far as we know, this familly has never been considered in a propositional base merging context. We have defined :

Given a pseudo-distance d, an integrity constraint μ, a profile $E = \{K_1, \ldots, K_n\}$, let ω be an interpretation. We define the distance $d_{d,Gmin}(\omega, E)$ as the list of numbers $(d_1, \ldots, d_n)$ obtained by sorting in increasing order the list of distances $\{d(\omega, K_i) \mid K_i \in E\}$. The models of $\triangle_{\mu}^{d,\text{GMIN}}(E)$ are the models of μ that are minimal with respect to the lexicographic order induced by the natural order.

Results : G_{min} operators

We get :

- rationality : G_{min} operators are majority merging operators, since it satisfies all the logical properties and majority.
- complexity : higher than quota operators, since the problem of knowing if a formula is a consequence of the merged base is in Δ_2^p.
- strategy-proofness : G_{min} operators are not strategy-proof in general, but only under restrictions, for example if the bases are complete.
- discrimination power : better than quota operators and formula-based merging operators.

G_{min} operators are slightly more complex and not strategy-proof in the general case, but they are fully rational and much more discriminating. They also lead to merged bases implying the disjunction of the bases from the considered profile, thus offering an interesting alternative to syntax-based merging operators (which are typically at least as hard from the complexity point of view and satisfy less rationality postulates).

Related work : Social choice theory

Strategy-proofness is a central issue in social choice theory. In particular, there is the well-known :

- ▷ Gibbard-Satterthwaite theorem : When at least three alternatives are possible, every social choice function (associating an alternative to a profile of rankings) which is onto and strategy-proof is dictatorial.

Many differences with our work exist :

- ▷ The preference relation considered in the merging are two-strata total pre-orders, and not strict orderings, and the result of the merging is still a two-strata total pre-order, and not a single world.
- ▷ The notion of strategy-proofness in the merging is more complex to define.
- ▷ The result is “universal” for the social choice functions, whereas there are some strategy-proofness merging operators.

Related work

- ▷ A study of strategy-proofness of some merging operators has been carried out by Meyer, Ghose and Chopra in 2001. The framework considered in this paper is clearly distinct from the one used in our work : agents may report full preference relations, encoded as stratified belief bases. The merging operators under consideration escape Gibbard-Satterthwaite theorem (as well as Arrow theorem) since a commensurability assumption between the agents' preference relations is made. Roughly, commensurability means that we allow to compare the satisfaction degrees of different agents. It means that we do not work with pre-orders, but with a more quantitative framework, where one uses a common (or at least comparable) scale for all agents.
- ▷ Links with multi-criteria decisions making

Futur thesis work

- Other indexes : for example, “Dalal index”. The idea behind that index is to define a satisfaction index from the Hamming distance between the agent’s beliefs and the merged base. This leads to a more gradual index, in particular when the merged base and the agent base are incompatible. An other idea is to take into account the satisfaction of other agents : an agent can be interested in satisfying her personal choices, but as well in satisfying some agents’s choices, or in harming some agents.
- Complexity : the manipulability of an operator is not a problem if finding out a strategy is hard. We are interested in “theorical” complexity, that is complexity of finding a strategy in the worst case, but also in “practical” complexity, that is finding the proportion of manipulable profiles for different merging operators.

Futur thesis work

- Coalitions : another interesting issue is to study the strategy-proofness problem when coalitions are allowed. The question is to know if a group of agents can coordinate for achieving a better result of the merging for all of them. This interesting issue requires more hypotheses on the agents abilities, since it requires communication abilities, in order to allow agents to propose to others to form a coalition, and to coordinate on the base each member of the coalition must give for achieving the wanted result. For this work, it seems that games in coalitional form, studied in game theory, can provide some interesting notions and results.

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