m4 - trying the MCP NMR retail - ver.3

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In this version we simplify all that is possible to simplify, down to the circumscribing of action, and the solution to the frame problem. We will try to make everything simple enough to prove everything that we want to prove (success with 3,3 and failure with 4,4), in a simple manner. No elaborations are treated here.

1 Formulas

The following formulas describe the relations and objects in the basic story.

1.1 Basic Formulas (mainly action related)

1.1.1 Language

Our language will include the following object constants: M,C,$S0$, $Situations$,$Boat$,$Banks$, which will be the sets of Missionaries and Cannibals, the first Situation, the set of Situations, the Boat, and the set of Banks, respectively. We need all of these to define the sorts of objects that we are dealing with (esp. to make sure that functions and predicates act on the right sets of objects). We will define the function and relation constants with their associated axioms in 1.3.3.

In this version of the paper, we restrict ourselves to actions of the form

$$A(group, bank) = using(Boat, go(group, bank))$$

Notice that the Boat is fixed, and we therefore implicitly assume that the boat is needed, that there is exactly one, and that there is no more actions possible. We further restrict the set of actions to be

$$Actions(s) = \left\{ A(group, bank) \mid \begin{array}{l} bank \in Banks \wedge \\ group \subset inhabitants(opp(bank), s) \end{array} \right\} \qquad (1)$$

All of the notions used in the above definitions are defined below.

1.1.2 can

Let A be a macro for $A(group, bank)$.

$$
\begin{array}{l}
\neg abq(A,s) \rightarrow can(A,s) \\
can(A,s) \rightarrow ok(s) \\
can(A,s) \rightarrow 0 < card(group) \\
can(A,s) \rightarrow (\forall p \in group)(location(p,s) = opp(bank)) \\
can(A,s) \rightarrow (\forall p \in group)(location(p, result(A,s)) = bank) \\
can(A,s) \rightarrow location(Boat, result(A,s)) = bank \\
can(A,s) \rightarrow card(group) \leq capacity(Boat) \\
can(A,s) \rightarrow water\text{-}crosser(Boat) \wedge available(Boat, group, s) \\
can(A,s) \rightarrow A \in Actions(s)
\end{array}
\tag{2}
$$

1.1.3 other axioms

The important part (can) relied on the following definitions and axioms.

$$
\begin{array}{l}
\{Lb, Rb\} = Banks \wedge Lb \neq Rb \wedge opp(Lb) = Rb \wedge opp(Rb) = Lb \\
isboat(Boat) \wedge capacity(Boat) = 2. \\
isboat(boat) \rightarrow water\text{-}crosser(boat) \\
available(means, group, s) \iff \\
\qquad isboat(means) \wedge group \subset inhabitants(location(means, s), s)
\end{array}
\tag{3}
$$

1.1.4 Goals

In the standard case of MCP, we should be able to infer

$$
\begin{array}{l}
location(grp, s) = bank \wedge location(boat, s) = bank \wedge ok(s) \wedge \\
isboat(boat) \wedge 0 < card(grp) \leq capacity(boat) \\
\rightarrow can(using(boat, go(grp, opp(bank))), s).
\end{array}
\tag{4}
$$

Notice that the $location(grp, s)$ is not well defined. We might want to use a predicate instead of a function.

This will involve some nonmonotonic reasoning. Up till now, we got (translating $A \in Actions(s)$ into its equivalent extensional form)

$$
\begin{array}{l}
can(using(Boat, go(group, bank)), s) \iff \\
\qquad bank \in Banks \wedge \\
\qquad card(group) \leq capacity(Boat) \wedge \\
\qquad water\text{-}crosser(Boat) \wedge available(Boat, group, s)) \wedge \\
\qquad ok(s) \wedge \\
\qquad 0 < card(group) \wedge \\
\qquad (\forall p \in group)(p \in M \cup C \wedge location(p,s) = opp(bank))
\end{array}
$$

Notice that

$$\begin{array}{l} (\forall p \in group)(location(p, result(A, s)) = bank) \\ location(Boat, result(A, s)) = bank \end{array} \tag{5}$$

are no longer prerequisites of the action (as it was the situation in the previous version of this paper), since we vary result. Of course we neglected to mention the rest of the framework of the circumscription, and the next sub-section deals with that.

Putting the last equivalence in an *easier to compute and handle* manner, it is equivalent to

$$\begin{array}{l} can(using(Boat, go(group, bank)), s) \iff \\ \qquad bank \in Banks \wedge \\ \qquad 0 < card(group) \leq capacity(Boat) \wedge \\ \qquad isboat(Boat) \wedge ok(s) \wedge \\ \qquad location(Boat, s) = opp(bank) \wedge \\ \qquad (\forall p \in group)(p \in M \cup C \wedge location(p, s) = opp(bank)) \end{array} \tag{6}$$

1.1.5 The Nonmonotonic framework

This section takes the nonmonotonic approach to formalizing action, as proposed by McCarthy in [McC86]. It works in this scenario, although it has some known weaknesses (cf [SS94])[1] We will use the following theory:

$$\begin{array}{l} FrameFunction = \{location(obj) \mid obj \in M \cup C \cup \{Boat\}\} \\ \forall f \in FrameFunction\ \forall s \in Situations\ \forall A \in Actions(s) \\ \qquad (\neg ab(f, A, s) \implies f(s) = f(result(A, s))) \end{array} \tag{7}$$

The reason for not including $can(A, s)$ in the antecedent of our inerita law will become apparent in our proof for lemma 1.1. We choose the following circumscription policy as our nonmonotonic inference tool:

$$\begin{array}{l} \textbf{Circ}\ abq\ \textbf{var}\ Holds, ab, can \\ \textbf{Circ}\ ab\ \textbf{var}\ Holds \end{array} \tag{8}$$

Notice that we did not mention *Holds* in our formalism, and we turn to it here as a shorthand for varying all the fluents. The motivation behind the above policy is to make as many actions possible, even in the price of making more changes result of actions [2].

[1] It is my belief that there is a theorem relating John's approach and Baker's approach. I have yet to think about it. The hypothesis says that any model that is possible in Baker's approach is also possible in John's approach.

[2] actually the order does not matter in this case, since we can put the circumscirption policy to force circumscribing ab before abq, as long as we keep can varying only for the circumscription of abq. We do prefer to do that the way we do, since it seems to make more sense, and we hope to generalize it in the future framework

Lemma 1.1 *The Circumscription policy (8) implies*

1.

$$\begin{aligned}&\forall s \in Situations\ \forall A \in Actions(s)\ (can(A,s) \implies \\ &\qquad (\forall p \in group)(location(p, result(A,s)) = bank) \wedge \\ &\qquad location(Boat, result(A,s)) = bank \wedge \\ &\qquad (\forall p \in (M \cup C \setminus group))(location(p, result(A,s)) = location(p,s)))\end{aligned}$$

2.

$$\begin{aligned}&\forall s \in Situations\ \forall A \in Actions(s) \\ &((\exists p \in (M \cup C \cup \{Boat\})(location(p, result(A,s)) \neq location(p,s))) \implies \\ &\qquad can(A,s))\end{aligned}$$

PROOF For the first part, notice that the first two rows of the succedent follow directly from (2). The third row is proved true given $can(A,s)$, using the persistence axiom (7). Assume that $\exists p \notin group, p \neq Boat$ s.t., $location(p, result(A,s)) \neq location(p,s)$. For this p we have no axiom refering to its location, with regard to the specific application of A to s (since it is neither the $Boat$ nor a member of $group$). Thus we can vary $location(p, result(A,s)) = location(p,s)$, and get a smaller ab, contradicting the fact that ab is minimal.

For the second part, let $s \in Situations, A \in Actions(s)$, and assume that $\neg can(A,s)$, but the location of $p \in M \cup C \cup \{Boat\}$ changed after applying A to s. Notice that since $\neg can(A,s)$ holds, all the sentences having $can(A,s)$ as an antecedent, are automatically tautologies. Thus, when we change $location(p, result(A,s))$ to be $location(p,s)$, the validity of (2) is preserved, and since we are allowed to vary $Holds$, we may change the values of $available$ (3) and ok (9), both relate to the location of some objects, and still preserve validity. Thus we can allow ab smaller than the one we got with $location(p, result(A,s)) \neq location(p,s)$ (and notice that the corressponding value for abq does not change). Contradiction. ■

1.2 Counting the Missionaries and Cannibals

The only formulas we need for our previous definitions/formulas are

$$\begin{aligned}&inhabitants(bank, s) = \{x \in M \cup C \mid location(x,s) = bank\} \\ &m(s, bank) = card(\{m \in M | location(m,s) = bank\}) \\ &c(s, bank) = card(\{c \in C | location(c,s) = bank\}) \\ &ok(s) \equiv \neg(0 < m(s, Lb) < c(s, Lb)) \wedge \neg(0 < m(s, Rb) < c(s, Rb)) \wedge s \in Situations\end{aligned} \tag{9}$$

The rest will be useful for the proofs (section 2):

$$
\begin{array}{l}
m(group) \equiv card(group) = 1 \wedge group \subset M \\
mm(group) \equiv card(group) = 2 \wedge group \subset M \\
c(group) \equiv card(group) = 1 \wedge group \subset C \\
cc(group) \equiv card(group) = 2 \wedge group \subset C \\
mc(group) \equiv card(group) = 2 \wedge (\exists x)(x \in group \cap M) \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad \wedge (\exists x)(x \in group \cap C)
\end{array} \tag{10}
$$

maximized,

1.3 The Frame Problem

We would like to conclude that if an action of going, or using the boat to go, is feasible, we maintain the restrictions over the relative quantities of missionaries and cannibals. But we only made half a step. We count them, and make some restrictions on their locations. By using fixed sets of Missionaries and Cannibals (M,C), and fixed sets of all the other objects under consideration, we solve many possible frame problems, such as humans appearing or disappearing suddenly. The following section deals with these *domain axioms* and the *initial situation*.

1.3.1 Domain axioms

$$
\begin{array}{l}
card(M) = 3 \\
card(C) = 3
\end{array} \tag{11}
$$

1.3.2 The initial situation $S0$

$$
\begin{array}{l}
S0 \in Situations \\
m(S0, Lb) = M \\
c(S0, Lb) = C \\
location(Boat, S0) = Lb
\end{array} \tag{12}
$$

1.3.3 Axioms regarding the functions and relations

$$
\forall s \in Situations\ \forall a \in Actions(s)\ (result(a, s) \in Situations) \tag{13}
$$

2 Proofs about the resulting formalism

2.1 A Proof that (3,3) works

Let us first see what happens when we want to move the first two cannibals. Let CC be a group of two cannibals in $S0$ (such that $cc(CC)$ holds).

2.1.1 The first step - $can(using(Boat, go(CC, Rb), S0))$

Using (6):

$$\left.\begin{array}{ll} Rb \in Banks & (3) \\ 0 < card(CC) \leq capacity(Boat) & (3)+(10) \\ isboat(Boat) & (3) \\ ok(S0) & (9)+(11)+(12) \\ location(Boat, S0) = opp(Rb) & (3)+(12) \\ (\forall p \in CC)(p \in M \cup C \wedge location(p, S0) = opp(Rb)) & (3)+(10)+(12) \end{array}\right. \quad (14)$$

We are left to prove the existence of CC, but for $S0$ it follows from (11) and (10).

For the rest of the stages it will be more difficult and might require some more notations. One is refered to the PVS proof of the MCP (ver. 2.0 and up). It uses set theory and besides the problems involving the applications of extensionality and counting, it has the problem of "selecting" memebers from a set. We ignore these notational issues here, noticing that they are solvable, in the expense of complicated ad-hoc notations, that do not contribute to the understanding of the problem and its analysis.

2.1.2 The generic step

Let us carry one more step. Let a_1 be the action performed in the previous section, and let

$$S1 = result(a_1, S0)$$

According to lemma 1.1,

$$\begin{array}{l} \forall p \in CC \ (location(p, S1) = Rb) \\ location(Boat, S1) = Rb \\ \forall p \in (C \cup M \setminus CC) \ (location(p, S1) = Lb) \end{array} \quad (15)$$

Thus, when we need one cannibal $C1$ to go back, the action is enabled as before. The only changes in the justifications are:

$$\begin{array}{ll} ok(S1) & (9)+(11)+(15)+(13) \\ location(Boat, S1) = opp(Lb) & (3)+(15) \\ (\forall p \in C1)(p \in M \cup C \wedge location(p, S1) = opp(Lb)) & (3)+(10)+(15) \end{array} \quad (16)$$

The existence of the needed group of people at each stage is also guaranteed by (15) above.

2.2 Proving that (4,4) has no solution

Let us first notice that what we are looking for is a sequence of actions that will result in transfering 4 missionaries and 4 cannibals from the Lb to the Rb.

For this version, we will change 11 to say

$$\begin{array}{l} card(M) = 4 \\ card(C) = 4 \end{array} \tag{17}$$

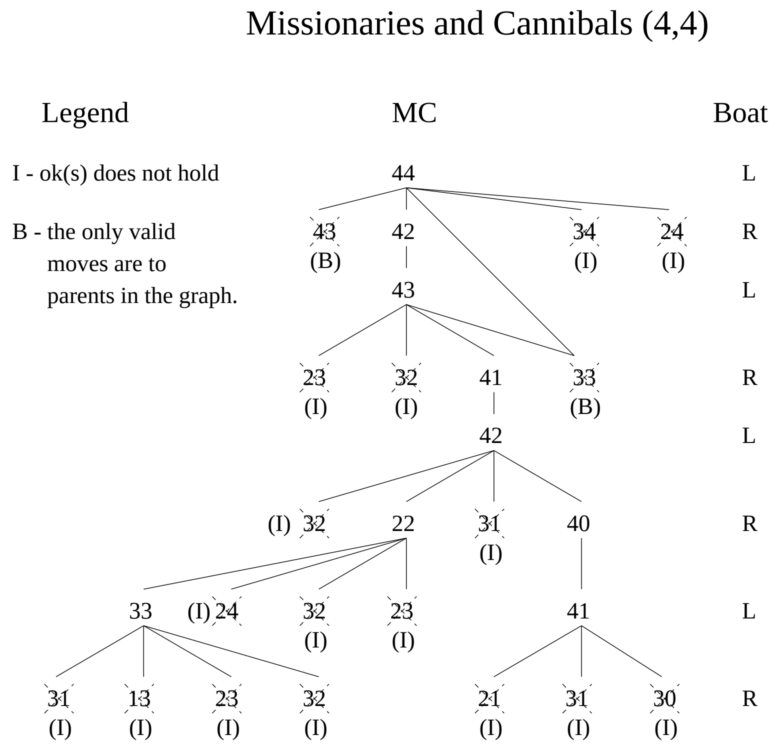
Now, let us assume that such a sequence Seq of length n exists. Notice, that because of lemma 1.1, any change in the location of any missionary or cannibal (or Boat) is governed by a *possible* action. Thus we have

$$1 \leq \forall i \leq n \ (can(a_i, s_{i-1}))$$

for $s_i = result(a_i, s_{i-1})$, and a_i the i-th element in Seq. Notice that there are at most 5 possible actions in every situation, *given that we do not distinguish among the missionaries or the cannibals that are located on the bank of the river* (We will justify this approach later in this section). That follows from our restrictions on can by $Actions(s)$, $location(Boat)$, and $capacity(Boat)$.

The following graph is a straightforward *unformal* proof that there is no such Sequence. The nodes are possible states represented in terms of the number of missionaries and cannibals on the left bank, and the location of the boat (Right or Left banks). Each load that is crossed out, is labeled with the reason for the crossing. There are some branches that are not mentioned, since they are obviously illegal.

Note that this is an undirected graph (although it is presented in a top-down manner). It formalizes the accessibility relation between situations. Since the actions are always retractable by the the appropriate conjugate action, there is always an action that will return us to the *parent* situation (parent node in the graph view).



In order to make that proof more formal we need to follow several steps. First, make sure that the possible actions and only those, are shown. This is pretty straightforward, as there are at most 5 possible actions possible in every situation (as argued above), and the preconditions that are situation/action dependent are on the cardinality of the group, on the location of the group (and the boat), and on the *ok*-ness of the situation.

Second we should make sure that the amount of people on the left bank, and the location of the boat, are the only issues that matter, in the current formalization. But as we just argued, the possible actions refer exactly to these properties of the selected group and the boat. The selection of specific members of the appropriate sets in each bank has no influence on their location (we picked the members of the set *from that bank*), or the set's cardinality.

Also, the ok-ness of the situation is dependent only on the cardinality of the appropriate sets on each bank, and the situation-ness of the situation (i.e., that s is actually a situation). Since the set of missionaries and the set of cannibals is constant (they are not situation dependent - (17)), and location is a complete function (on missionaries, cannibals and the boat), the cardinalities of the sets of missionaries and cannibals in the right bank is directly calculatable from the cardinalities of the relative sets in the left bank. Since the fact that s is a situation follows from the application of the previous action to a situation and (13), we conclude that the amount of missionaries and cannibals on each bank in the resulting situation, is dependent only on the relative amounts in the prior situation, the amounts of people from each set in the boat (the group that actually *goes*), and the location of the boat (all of that - given (1.1)).

After these formal notes, we can see that the above graph is an exact proof of the inaccessibility of (44) and (00), and thus an inaccessibility of the relative situations[3].

References

[McC86] John McCarthy. Applications of circumscription to formalizing common sense knowledge. *Artificial Intelligence*, 28:89–116, 1986. Reprinted in [McC90].

[McC90] John McCarthy. *Formalization of common sense, papers by John McCarthy edited by V. Lifschitz.* Ablex, 1990.

[SS94] Erik Sandewall and Yoav Shoham. Nonmonotonic temporal reasoning. In *Handbook of Logic in Artificial Intelligence and Logic Programming, Volume 4: Epistemic and Temporal Reasoning.* Oxford University Press, 1994.

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[3] A somewhat simpler and more intuitive proof than the graph might go along the lines of showing that every situation in which the two quantities (missionaries and cannibals on the left bank) are between 1 and 3, is illegal (not *ok*) (and thus there is no action possible in these situations). The graph seems to be more simple to comprehend, though.