CS323: Circumscription

- Two readings for today:
  - *Circumscription--A Form of Non-Monotonic Reasoning*. [McC80]
  - *Applications of Circumscription to Formalizing Common Sense Knowledge*. [McC86]
  - Optional, but good:
    - *Circumscription*. [Lif93]

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**Circumscription -- A Form of Nonmonotonic Reasoning: Motivation**

- Qualification problem: no way to specify exact conditions for performance of an action
- Solution to this is circumscription:
  - rule of conjecture used to jump to conclusions
  - assumes propositions false unless said otherwise (CWA assumption for databases)
  - the objects that are shown to have property P are the only ones.
Circumscription -- A Form of Nonmonotonic Reasoning: Examples

- There are 3 blocks: A, B, and C.
  (From that you assume that there are no other blocks)  Domain circumscription
- Onn, Stefan, and Lauren are all getting As in CS323.  Predicate circumscription
- Boats (can use to cross a river unless something prevents you), tools.

Circumscription -- A Form of Nonmonotonic Reasoning: What’s Monotonicity?

Monotonicity is a feature of most logical systems. Say A and B are sets of sentences, and q is a proposition:

if A |- q and A ⊆ B, then B |- q

(adding more sentences to the premises only increases the number of conclusions!)
Circumscription -- A Form of Nonmonotonic Reasoning: What’s Nonmonotonicity?

- So then nonmonotonicity is when: 
  \( A \models q \) and \( A \subseteq B \), but not necessarily \( B \models q \)
- For example,
  \( A = \{ \text{broken(boat), ..} \} \)
  \( q = \neg \text{cross(person)} \)
  \( B = A \cup \{ \text{spans(river, bridge)} \} \)
- Also applies to semantic notion (\( |\models| \))

Circumscription -- A Form of Nonmonotonic Reasoning

- Seems to be a rule of conjecture in life, but in puzzles a rule of inference
- Against probability/fuzzy logic:
  - probability of MCP problem not meaningful,
  - probability of bridge given that even less meaningful
  - people consider normal case first -- not the sample space of all possibilities
Circumscription -- A Form of Nonmonotonic Reasoning: Formalism

- Predicate Circumscription: minimize extent of predicate $P$
- Given a theory $A$, two ways to formalize:
  1. Semantic: Pick out the models of $A$ which minimize the extent of some domain or predicate
  2. Syntactic: Append a sentence to $A$ to do minimization syntactically.

Circumscription -- A Form of NMR: Predicate Circumscription

- Some notation first:
  - We abbreviate $P(x)$ for $P(x_1, \ldots, x_n)$
  - We define $\Phi \leq P$ as $\forall x. \Phi(x) \rightarrow P(x)$,
  - We also say $\Phi = P$ as $\forall x. \Phi(x) \equiv P(x)$
  - $A(\Phi)$ is theory $A$, with all occurrences of $P$ replaced with $\Phi$
- Magical circumscription of $P$ in $A(P) = \text{Circ}[A(P); P]$: $A(P) \land \forall \Phi. [(A(\Phi) \land \Phi \leq P) \rightarrow \Phi = P]$
Circumscription -- A Form of NMR: Predicate Circumscription

- Again:
  \[ \text{Circ}[A(P); P] = A(P) \land \forall \Phi . [(A(\Phi) \land \Phi \leq P) \rightarrow \Phi = P] \]

- Alternate, equivalent form:
  \[ \text{Circ}[A(P); P] = A(P) \land \neg \exists \Phi . (A(\Phi) \land \Phi < P) \]

- McCarthy in McC80 uses schemas rather than second-order logic. In McC86 back to second-order.

Circumscription -- A Form of NMR: Predicate Circumscription

- Can also do joint circumscription over two predicates P and Q. \[ \text{Circ}[A(P, Q); P, Q] = A(P, Q) \land \]
  \[ \forall \Phi, \Psi . [(A(\Phi, \Psi) \land \Phi \leq P \land \Psi \leq Q) \rightarrow (\Phi = P \land \Psi = Q)] \]

- Can also allow other symbols (predicates, constants, functions) to vary (more on this later).
Circumscription -- A Form of NMR: Examples of Predicate Circumscription

Simpler examples from Lif93:
1. A(P) is P(a):
2. A(P) is ¬P(a):
3. A(P) is P(a) ∧ P(b):
4. A(P) is P(a) ∨ P(b):
5. A(P) is P(a) → P(b):
6. A(P) is ∀x.Px:
7. A(P) is ∀x.Qx→ Px:

Circumscription -- A Form of Nonmonotonic Reasoning: Extensions

- Irrelevant symbols in circumscription get factored out:
  Say A(P) = B ∧ C(P).
  Circ[A(P); P]
  = A(P) ∧ ∀Φ.[(A(Φ) ∧ Φ ≤ P) → Φ = P ]
  = B ∧ C(P) ∧ ∀Φ.[(B ∧ C(Φ) ∧ Φ ≤ P) → Φ = P ]
  = B ∧ Circ[C(P); P]

- Domain circumscription:
  A ∧ ∀Φ.[Axiom(Φ) ∧ AΦ → ∀x.Φ(x)]
Circumscription -- A Form of NMR: Semantic Way of Circumscription

Model theory of circumscription:
- order models of $A$ by relation $\leq_p$, where two models $M(A) \leq_p N(A)$ iff
  (extension of $P$ in $M$) $\subseteq$ (extension of $P$ in $N$) and
  everything else same
- pick the $\leq_p$ - minimal models of $A$. (Those models $M(A)$ such that $\exists M'(A). M'(A) <_p M(A)$)
- $\text{Circ}[A(P); P]$ is satisfied in any of these minimal models.

Circumscription -- A Form of NMR: Varying Symbols

- From Lif93: (MCC86 refers to this in the bird example, so I should explain here.)
- Certain symbols can “vary” in the circumscription. We denote this as $\text{Circ}[A(P, Z); P; Z]$, where $Z$ is the relation/constant/function symbol to be varied.
- There are parallel semantic and syntactic ways of describing variation:
Circumscription -- A Form of NMR: Varying Symbols

• For Circ[A(P, Z); P; Z], we redefine our model ordering relation to be:
  - $M(A) \leq_P N(A)$ iff
    (extension of P in M) $\subseteq$ (extension of P in N) and
    everything else same except Z
• This means that we don’t have to worry about the Zs being the same in M and N, which means that they are allowed to vary.

Circumscription -- A Form of NMR: Varying Symbols

• Syntactically, we write the circumscription formula as:
  Circ[A(P, Z); P; Z] =
  $A(P, Z) \land \forall \Phi, \zeta[(A(\Phi, \zeta) \land \Phi \leq P) \rightarrow \Phi = P]$
• We can set $\zeta$ to be whatever we like, in order to help satisfy the LHS of the implication.
• Remember, Z can be a constant, function, or predicate symbol!
Circumscription -- A Form of NMR: Conclusions

- Circumscription is not a nonmonotonic logic, but a type of reasoning augmenting FOL
- More expressive than default logic
- Both McCarthy and Lifschitz propose using circumscription in a reasoning program, where it is described what predicates are circumscribed how.
- (obvious) circumscription may lead to different results, depending on how it is formalized.

Applications of Circumscription to Formalizing Common Sense Knowledge

- Introduces formula circumscription, where instead of minimizing the extent of a predicate, you minimize the extent of a formula:
  \[ A(P) \land \forall \Phi, [(A(\Phi) \land E(\Phi) \leq E(P)) \rightarrow E(\Phi) = E(P)] \]
- Instead of minimizing \( P \) directly (predicate circumscription), you minimize \( E(P) \).
- More expressive than just trying to minimize the extension of predicates
Applications of Circumscription: Seven Uses of an Elephant

1. Communication convention: If not mentioned, assume not/default
2. Database convention: only certain predicates CWAed
3. Rule of conjecture: “Most birds fly.”
4. Policy representation: “The meeting is on Wed. unless there is another decision.”
5. Streamlined version of probabilities
6. Auto-epistemic reasoning
7. Common sense physics and psychology

Applications of Circumscription: Abnormalities

- Filter in all the qualifications into one predicate, $ab \; z$, where $z$ represents some aspect of an object.
- $Ab \; z$ represents all the conditions that could make some property of an object not hold:
  \[
  \forall x. \neg ab(\text{aspect1}(x)) \rightarrow \neg \text{flies}(x)
  \]
  “Most objects do not fly.”
- Then, minimize $ab$. 
Applications of Circumscription

- Inheritance hierarchies
- UNH
- Nixon Diamond, Vancouver vs. Toronto
- General Is-a hierarchy treatment
- Blocksworld (changing color and location)
- More birds flying

Applications of Circumscription: Prioritized Circumscription

- Consider the ordering
  \[ ab \leq ab' \equiv \forall x. \ ab(x) \rightarrow ab'(x) \]
- For each aspect, we can define an ordering
  \[ ab \leq_i ab' \equiv \forall x. \ ab(\text{aspect}_i(x)) \rightarrow ab'(\text{aspect}_i(x)) \]
- Then we can have an ordering on the orderings, say on 1 and 2:
  \[ ab \leq_{1<2} ab' \equiv ab \leq_2 ab' \land [ab =_2 ab' \rightarrow ab \leq_1 ab'] \]
- This means to see if \[ ab \leq_{1<2} ab' \], check if the abs are ordered wrt to aspect$_2$. If they are equal wrt to 2, then fall back on aspect$_1$. 
Applications of Circumscription: Considerations and Remarks

- Certain forms of Circ[A(P); P] are collapsible to FOL.
- Circumscription can be viewed as a process of compiling higher-order logic into FOL.
- Circumscription is more computable than Reiter’s default logic.
- Still need to add hints to the reasoning program as to how to circumscribe.
- Still other undiscovered ways to use nonmonotonic reasoning.