CHAOS AND MOVING MARS TO A BETTER CLIMATE

John McCarthy
Computer Science Department
Stanford University
Stanford, CA 94305
jmc@cs.stanford.edu
http://www-formal.stanford.edu/jmc/

2007 Oct 6, 8:28 p.m.

Abstract

In a chaotic dynamical system, small deviations from given initial conditions produces large changes over time. This suggests that chaotic systems are controllable by making small changes initially. The solar system is a somewhat chaotic dynamical system, so maybe humanity can control its evolution to produce desired effects.

The specific idea of this article is to move the planet Mars to an orbit at the same distance from the sun as the earth's orbit. This would make Mars warmer, which might make it more habitable.

The scheme is to use a *tame asteroid* that makes several thousand encounters with Mars, Venus and Jupiter to exchange energy and angular momentum among these planets, thus moving Mars to the desired orbit. Conservation of both energy and angular momentum requires that two other planets besides Mars be used.

A *tame asteroid* is one that makes repeated encounters with planets that magnify small perturbations of its orbit. The object is to achieve the goal of moving Mars with minimal total Δv, i.e. minimal rocketry.

Moving Mars will take some tens of thousands of years, but not millions of years.[1]

1 Chaos and controllability

Consider a system governed by systems of differential equations that evolves chaotically. Thus small variations in initial conditions are amplified to produce large changes in later states. Suppose humans can make small purposeful changes. Then perhaps the later states can be controlled. Suppose some initial conditions for a system are specified, and we can compute the future history. Small deviations from the initial conditions grow exponentially with time. Thus making a large change with a very small expenditure of energy requires a long time, but the time grows only logarithmically with the inverse of the energy expended. We can ask what states are reachable by small changes in initial conditions.

Take the solar system as an example. There are certain integrals of the motion of the solar system as a whole, e.g. energy and angular momentum. It is clear that we cannot change the total energy of the system by more than the energy we have available to expend. However, we can in principle change the energy of one planet at the expense of energy of some others.

[2]

Angular momentum is also conserved, but it isn't obvious to me what the trade-off is between expending energy to change angular momentum. Taking a mass m a distance d from the sun allows making a change of angular momentum of $m\Delta vd$. If we want the changed angular momentum to be effective in the inner solar system, then Δv must not be so large that the mass escapes. The time for the mass to return to the inner solar system is proportional to $d^{\frac{3}{2}}$.

As a concrete problem, consider moving Mars to an orbit at the same

[1] This article is an elaboration of a web page (http://www-formal.stanford.edu/jmc/future/mars.html) put up for my students in a class in Technological Opportunities for Humanity at Stanford University. It may change.

[2] This conclusion depends on regarding the solar system as isolated. If we consider a large system involving Alpha Centauri and our own solar system, we can imagine increasing our solar system's energy by taking energy from the Alpha Centauri system. It seems apparent that this would take a very long time. If it were determined to take more than (say) 10^{100} years, then we could regard the energy of our own system as essentially unchangeable. Actually it might take only tens or hundreds of thousands of years.

distance from the sun as the earth, keeping it on the other side of the sun from the earth to eliminate gravitational interaction with the earth. This would make Mars warmer, which would facilitate human settlement.

2 A tame asteroid

The idea is to use an asteroid to transfer energy and angular momentum among Mars, Jupiter and Venus in a way that makes Mars move closer to the sun. We would expect Venus and Jupiter to move further away, but not much. We'll see how the mathematics turns out. We hope to do this with only incidental Δv applied to the asteroid by rockets.

The largest asteroid is Ceres, which has a mass about $\frac{1}{1{,}000}$th that of Mars. The most favorable encounter of Ceres with Mars is when Ceres comes as close to Mars as possible and has its velocity reversed in the Mars-centered coordinate system. The Δv of Ceres is then approximately the escape velocity from Mars. [We'll give the details later]. The Δv given to Mars is then $\frac{1}{1{,}000}$ of this escape velocity. Actually, the encounters will not usually be so favorable, so there will be some efficiency factor less than 1.0. The escape velocity from Mars is about 5,100 meters per second, so a Δv for Mars of 24 km/sec would require at least 4,800 encounters. If we imagine that the orbital period of an asteroid within the orbit of Jupiter is about 10 years, then we are talking about some multiple of 48,000 years. Very likely, our descendants will become interested in starting such a project only after they have had a technological civilization for some tens of thousands of years.

Ceres is the largest asteroid, but it looks like would be very expensive to get Ceres out of its very stable orbit. Therefore, we propose using an asteroid from the Kuiper belt which would be much more easy to manipulate at first. (This idea belongs to (DGK01).) The biggest of these isn't known yet, but we can imagine it to be $\frac{1}{10}$th the mass of Ceres.

We want a tame asteroid.

Asteroids large enough to have significant gravitational effects are expensive to move much with feasible rockets. Therefore, we want an asteroid orbit that passes by many planets. Passing close to a planet should amplify any perturbation of the orbit. The earlier the perturbation can be made the greater the amplification. Note that you get no amplification out of a full Keplerian ellipse about the sun, because the ellipical orbit amounts to a focussing effect. A small perturbation now will cause a slightly changed ellipse that will repeat itself unless there is a further perturbation.

The one effect that does grow with time in a perturbed elliptical orbit is the time change ΔT of arrival at a given point. If the period is changed, then the perturbation will grow linearly with time. This effect can be used to adjust the phase of the next encounter of the asteroid with the orbit of the planet. It looks like by waiting long enough we can achieve the next encounter with an arbitrarily small Δv of the asteroid.

The first mathematical question is whether there can be a tame asteroid. Ideally its orbit would, if unperturbed, graze planets for the indefinite future. Let's call this a tame orbit. The second question is whether an early enough small perturbation could make the asteroid switch to a different tame orbit. If so, the third question is what is the set of tame orbits reachable from an initial orbit. As a first approximation to these questions we regard the asteroid as having infinitesimal mass, so the planets themselves are not perturbed.

The next main question arises when the finite mass of the asteroid is taken into account. What class of planetary configurations can be reached with an arbitrarily small tame asteroid? Can these configurations be reached with an arbitrarily small Δv given enough time?

The specific question is whether an asteroid from the Kuiper belt can be tamed and used to move Mars to an orbit on the opposite side of the sun at the earth's distance from the sun.

3 Conservation of Energy and Angular Momentum

Since the asteroid is small compared to planets, and our goal is to directly apply very little total Δv for the asteroid, the main effect is exchange of energy and angular momentum among the three planets.

Before trying to design orbits for the tame asteroid, we compute the changes in the orbits of Jupiter and Venus required to move Mars to one AU from the sun. We assume that energy and angular momentum are conserved, i.e. that the asteroid itself overall contributes nothing, because of its small size.

Here are some equations in which we assume that the planets have circular orbits. We derive relations between the energy $\mathcal{E}$, the angular momentum h and the distance of a planet from the sun. We use the speed v of the planet in its orbit as an intermediate variable.

We start with equations for a single planet in a circular orbit.

$$\mathcal{E} = \frac{1}{2}mv^2 - \frac{m\mu_\odot}{r} \tag{1}$$

$$h = mvr, \text{assuming circularity} \tag{2}$$

$$\frac{v^2}{r} = \frac{\mu_\odot}{r^2}, \text{ and so} \tag{3}$$

$$v = \sqrt{\frac{\mu_\odot}{r}} \tag{4}$$

Circularity also gives

$$\mathcal{E} = \frac{1}{2}m\frac{\mu_\odot}{r} - m\frac{\mu_\odot}{r} = -\frac{m\mu_\odot}{2r} \tag{5}$$

$$h = mvr = m\sqrt{\mu_\odot r}, \text{ and so} \tag{6}$$

$$r = \frac{h^2}{m^2\mu_\odot} \tag{7}$$

and finally

$$\mathcal{E} = -\frac{m\mu_\odot}{2r} = -\frac{m\mu_\odot}{2\frac{h^2}{m^2\mu_\odot}} = -\frac{m^3\mu_\odot^2}{2h^2}. \tag{8}$$

Having expressed $\mathcal{E}$ in terms of h, we are ready to write the conservation laws for the total energy $\mathcal{E}$ and the total angular momentum h. We use subscripts for the quantities associated with the three planets.

Conservation of energy for the three planets, e.g. Mars, Venus and Jupiter gives for the new configuration

$$\mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_3 = \mathcal{E}, \tag{9}$$

where $\mathcal{E}$ is the original total energy.

Conservation of angular momentum gives for the new configuration

$$h_1 + h_2 + h_3 = h, \tag{10}$$

where h is the original total angular momentum.

We assume that planet 1, e.g. Mars, is to be moved to a desired circular orbit, and therefore we know $\mathcal{E}_1$ and h_1. We need to solve for $\mathcal{E}_2$, $\mathcal{E}_3$, h_2, and h_3. We use (8) to express the energies in terms of the angular momenta, so (9) then becomes

$$\frac{m_1^3\mu_\odot^2}{2h_1^2} + \frac{m_2^3\mu_\odot^2}{2h_2^2} + \frac{m_3^3\mu_\odot^2}{2h_3^2} = \mathcal{E}. \tag{11}$$

(10) and (11) must be solved for h_2 and h_3, which will then allow determining r_2 and r_3 by substituting in (7).

These equations have the form

$$\begin{array}{c} x + y = a \\ \frac{c}{x^2} + \frac{d}{y^2} = e. \end{array} \tag{12}$$

Substituting $y = a - x$ gives

$$\frac{c}{x^2} + \frac{d}{(a-x)^2} = e. \tag{13}$$

Solving these equations for moving Mars to 1.0 AU with the aid of Venus and Jupiter yields

new Venus distance = 1.99886e+09 compared to 1.07700e+11

new Jupiter distance = 7.79062e+11 compared to ¿7.78000e+11

Venus comes out distressingly close to the sun, whose radius is 6.96000e+08. Oh well, nobody we know lives on Venus.

All distances are in meters.

4 A scheme with minimal total Δv

Consider an asteroid that starts very far out. The further the better as concerns energy spent on adjusting the orbit of the asteroid, but the further out the longer the whole process will take. The asteroid makes one encounter with a planet per trip into the inner solar system. On successive trips it encounters Mars, Venus and Jupiter but not necessarily in a fixed order. The encounters are on the correct side to get the sign of the Δv of the planet correct. The distance of the encounter from the planet is large enough so that the asteroid will come back out again after the encounter. The successive encounters arranged so that energy and angular momentum are transferred in the correct proportions in two senses. The first sense is that the asteroid should always have its perihelion inside the orbit of Venus and its aphelion way out. The second sense is that energy and angular momentum are exchanged in the correct ratio among the three planets. Because of conservation of energy and angular momentum, these two conditions will agree.

When the asteroid is very far out very little Δv is required to adjust the phase of its next planetary encounter. In the limit of the asteroid going to infinity, zero Δv is required. Because there is only one encounter per trip, only one condition has to be satisfied. The adjustment can take into account the inclination of the orbit of the planet.

5 Differential equations

It is interesting to see how r_2 and r_3 vary as r_1 is varied but keeping the total energy and the total angular momentum of the three planets constant. As usual, we preserve circular orbits.

Conservation of energy gives

$$\begin{aligned} 0 &= d\mathcal{E} \\ &= d\mathcal{E}_1 + d\mathcal{E}_2 + d\mathcal{E}_3 \\ &= d(\frac{\mu_\odot m_1}{2r_1}) + d(\frac{\mu_\odot m_2}{2r_2}) + d(\frac{\mu_\odot m_3}{2r_3}) \\ &= \frac{\mu_\odot}{2}(m_1 r_1^{-2} dr_1 + m_2 r_2^{-2} dr_2 + m_3 r_3^{-2}) dr_3). \end{aligned}$$

Thus

$$m_2 r_2^{-2} dr_2 + m_3 r_3^{-2}) dr_3 = -m_1 r_1^{-2} dr_1. \tag{14}$$

Conservation of angular momentum gives

$$\begin{aligned} 0 &= dh \\ &= dh_1 + dh_2 + dh_3 \\ &= d(m_1 \mu_\odot r_1{}^{\frac{1}{2}}) + d(m_2 \mu_\odot r_2{}^{\frac{1}{2}}) + d(m_3 \mu_\odot r_3{}^{\frac{1}{2}} \\ &= \frac{1}{2}\mu_\odot^{\frac{1}{2}}(m_1 r_1^{-\frac{3}{2}} dr_1 + m_2 r_2^{-\frac{3}{2}} dr_2 + m_3 r_3^{-\frac{3}{2}} dr_3) \end{aligned}$$

Thus

$$m_2 r_2^{-\frac{3}{2}} dr_2 + m_3 r_3^{-\frac{3}{2}} dr_3) = -m_1 r_1^{-\frac{3}{2}} dr_1. \tag{15}$$

We can solve (14) and (15) for dr_2 and dr_3, giving

$$dr_2 = -\frac{m_1}{m_2}\frac{r_1^2 - r_1^{-\frac{3}{2}} r_3^{-\frac{1}{2}}}{r_2^{-2} - r_2^{-\frac{3}{2}} r_3^{-\frac{1}{2}}} dr_1. \tag{16}$$

Since the 2 and the 3 subscripts play symmetrical roles, we have

$$dr_3 = -\frac{m_1}{m_3}\frac{r_1^2 - r_1^{-\frac{3}{2}} r_2^{-\frac{1}{2}}}{r_3^{-2} - r_3^{-\frac{3}{2}} r_2^{-\frac{1}{2}}} dr_1. \tag{17}$$

6 Remarks and Acknowledgments

Consider the possibility that introducing an arbitrarily small tame asteroid could throw a planet out of the solar system. This would be a different kind of instability than those that have been so far considered. It would be of mathematical interest to prove that a system consisting of a sun and three planets could be disrupted by a single arbitrarily small tame asteroid.

Notes:

1. Another way would use multiple asteroids from the Kuiper belt or even the Oort cloud. Maybe no Jupiter or Venus, and each asteroid makes a single pass and is kicked out of the solar system. Many can be en route at once. I suspect several times the mass of Mars will be required. specifically in the ratio of the escape velocity from a solar orbit at Mars distance from the sun to the Mars escape velocity. A single Jupiter pass may improve the numbers.

2. It looks like a lot can be worked out by considering single collisions. For example, suppose an asteroid from far out has an elastic collision with Mars. It will take energy and angular momentum from Mars. I suppose that if it were to collide again with Mars it would tend to give back what it took. How does this modify if we have an intermediate collision with Jupiter or Venus? In the end we need both, but let's see what happens with one.

3. It may be a lot easier to move Mars than the above considerations suggest. The solar system is a lot more chaotic, even in the short term than was thought 10 years ago. I found (PC05) very informative about exotic orbits as applied to spacecraft and as observed and calculated for the short period comet Oterma. Whether this would help move Mars would require

difficult mathematics and computations, at least by present standards. Our descendants my find the matter obvious.

Tom Costello commented usefully on an early version of this article. I have learned from (DGK01), and its ideas have triggered further thinking, as has discussion with Don Korycansky and Greg McLaughlin. R. William Gosper used Macsyma to solve the quartic equations.

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