The Mutilated Checkerboard in Set Theory

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An 8 by 8 checkerboard with two diagonally oppo
squares removed cannot be covered by dominoes e
of which covers two rectilinearly adjacent squares.
present a set theory description of the proposition
an informal proof that the covering is impossible. W
no present system that I know of will accept either
formal description or the proof, I claim that both sho
be admitted in any *heavy duty set theory*.

We have the definitions

$$Board = Z8 \times Z8,$$

$$mutilated\text{-}board = Board - \{(0,0),(7,7)\},$$

$$domino\text{-}on\text{-}board(x) \equiv (x \subset Board) \wedge card(x) = 2$$
$$\wedge (\forall x1\ x2)(x1 \neq x2 \wedge x1 \in x \wedge x2 \in x$$
$$\supset adjacent(x1, x2)),$$

$$
\begin{aligned}
&adjacent(x1, x2) \equiv |c(x1, 1) - c(x2, 1)| = 1 \\
&\wedge c(x1, 2) = c(x2, 2) \\
&\vee |c(x1, 2) - c(x2, 2)| = 1 \wedge c(x1, 1) = c(x2, 1),
\end{aligned}
$$

and

$$
\begin{aligned}
&partial\text{-}covering(z) \\
&\equiv (\forall x)(x \in z \supset domino\text{-}on\text{-}board(x)) \\
&\wedge (\forall x\ y)(x \in z \wedge y \in z \supset x = y \vee x \cap y = \{\})
\end{aligned}
$$

Theorem:

$$\neg(\exists z)(\textit{partial-covering}(z) \wedge \bigcup z = \textit{mutilated-board})$$

Proof:

We define

$$x \in Board \supset color(x) = rem(c(x,1) + c(x,2), 2)$$

$$domino\text{-}on\text{-}board(x) \supset (\exists u\ v)(u \in x \wedge v \in x \wedge color(u) = 0 \wedge color(v) = 1),$$

$$partial\text{-}covering(z) \supset card(\{u \in \bigcup z | color(u) = 0\}) = card(\{u \in \bigcup z | color(u) = 1\}),$$

$$card(\{u \in mutilated\text{-}board | color(u) = 0\}) \neq card(\{u \in mutilated\text{-}board | color(u) = 1\}), \quad ($$

and finally

$$\neg(\exists z)(\mathit{partial\text{-}covering}(z) \ \wedge \ \mathit{mutilated\text{-}board} = \bigcup z) \ ($$

Q.E.D.