CONCEPTS AS OBJECTS

John McCarthy
Computer Science Department
jmc@cs.stanford.edu
http://www-formal.stanford.edu/jmc/

1. Concepts (including propositions) as objects

2. Functions from objects to concepts of them.

3. Concepts and propositions are not a natural kind.

   - There are a variety of useful spaces of concept

   - Concepts are (usually) approximate entities.

Concepts and propositions—1

"...it seems that hardly anybody proposes to use differ
variables for propositions and for truth-values, or dif
ent variables for individuals and individual concepts.
(Carnap 1956, p. 113).

Variables for propositions and individuals are writter
lower case, e.g. p and x. Variables for propositions
individual concepts are capitalized, e.g. P and X.

This talk is about expressiveness rather than for prese
ing a theory.

Concepts and propositions–2

We write

$denotes(Mike, mike)$ or when functional, $mike = deno$

$Telephone(Mike)$ is the concept of Mike's telephone

$denot(Telephone(Mike)) = telephone(mike)$

Knowing what and knowing that

$knows(pat, Telephone(Mike))$

Suppose $telephone(mike) = telephone(mary)$

$Telephone(Mike) \neq Telephone(Mary)$

Possibly, $\neg knows(pat, Telephone(Mary))$

Truth values and propositions:
$man(mike)$
$true(Man(Mike)$
$knows(pat, Man(Mike))$ means Pat knows whether M
Possibly $knows(pat, Man(Mike)) \wedge \neg man(mike)$

$k(pat, Man(Mike)) \equiv true(Man(Mike)) \wedge knows(pat, M$

Equality and Existence

$true(Telephone(Mike)\ EqualsC\ Telephone(Mary)$, alth

$Telephone(Mike) \neq Telephone(Mary)$

$telephone(denot(Mike)) = telephone(denot(Mary))$

$telephone(mike) = telephone(mary)$

$denot(Telephone(Mike)) = denot(Telephone(Mary))$

$$(\forall X)(exists(X) \equiv (\exists x)denotes(X, x))$$

$ishorseC Pegasus$

$Winged(Pegasus)$

?$true$(Winged-Horse($Pegasus$))

$true$(Greek mythology, Winged-Horse($Pegasus$))

$\neg exists(Pegasus)$

We can have

$$(\exists X)(exists(\text{Greek Mythology}, X) \wedge \text{Winged-Horse}(X$$

but most likely, there doesn't have to be a domain
Greek mythological objects. This suggests that some
the rules of inference of predicate logic be weakened
such theories.

About propositions

$$true(Not(P)) \equiv \neg true(P)$$
$$true(P \ And \ Q) \equiv true(P) \wedge true(Q)$$
? $P \ And \ Q = Q \ And \ P$
? $P \ And \ (Q \ Or \ R) = (P \ And \ Q) \ Or \ (P \ And \ R)$

This way lies NP-completeness and even undecidablity whether two formulas name the same proposition.

Functions from things to concepts

Numbers can have standard concepts $Concept1(n)$ i certain standard concept of the number n. Writing Co suggests that there might be another mapping $Conc$ from numbers to concepts of them.

We can have

$\neg knew(kepler, CompositeC(Number(Planets)))$,
and also
$knew(kepler, (CompositeC(Concept1(denot(Number(Pl$

Functions from things to concepts–2

Russell's example: *I thought your yacht was longer th* *it is.* can be treated similarly, although it requires a fu tion going from the concept $Length(Youryacht)$ to w I thought its value was.

$$denot(I, Length(Youryacht)) > length(youryacht)$$

Functions from things to concepts–3

We may also want a map from things to concepts of th in order to formalize a sentence like, *"Lassie knows location of all her puppies"*. We write this

$$(\forall x)(ispuppy(x, lassie) \supset knowsd(lassie, LocationdC(Co$$

$Conceptd$ takes a puppy into a dog's concept of it, $Locationd$ takes a dog's concept of a puppy into a do concept of its location. The axioms satisfied by $know$ $Locationd$ and $Conceptd$ can be tailored to our ideas what dogs know.

$$(\exists n2)(k(pat, Concept2(n2)\ EqualsC\ Telephone(Mike)$$
$$\equiv knows(pat, Telephone(Mike))$$

or

$$knows(pat, Telephone(Mike))$$
$$\equiv denot(pat, Telephone(Mike)) = telephone(mike$$

Concepts as approximate entities

• Approximate entities occur in human common se
reasoning. They don't have if-and-only-if definitions,
the rock and ice constituting Mount Everest.

• The set of individual concepts of Greek mytholog
another approximate entity. Few of them have deno
tions.

• The logical way of handling approximate entities is
axiomatize them weakly. Did Pegasus have a mother

• $exists(\text{Greek Mythology}, Pegasus)$,
$\neg exists(\text{Greek Mythology}, Thor)$,
$\neg exists(\text{Greek Mythology}, \text{George Bush})$,
$exists(\text{Greek Mythology}, Mother(Pegasus))$?

Mr. S and Mr. P

Two numbers m and n are chosen such that $2 \leq m$ … $n \leq 99$. Mr. S is told their sum and Mr. P is told t… product. The following dialogue ensues:

Mr. P: I don't know the numbers.
Mr. S: I knew you didn't know. I don't know either.
Mr. P: Now I know the numbers.
Mr S: Now I know them too.

In view of the dialogue, what are the numbers?

"Two puzzles involving knowledge"
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Formalizing Mr. S and Mr. P

$knows(person, pair, time), k(person, Proposition, time$

persons: s, p, sp

$\neg knows(p, Pair0, 0)$

$knows(s, Sum(Pair0), 0)$

$knows(p, Product(Pair0), 0)$

$$(\forall pair)(sum(pair) = sum(pair0)$$
$$\rightarrow \neg k(s, Not(Pair0\ Equal\ Concept1(pair)), 0))$$

$k(sp, \ldots, 0)$

In the paper $A(w1, w2, person, time)$ means that in w
$w1$, world $w2$ is possible for $person$ at $time$.

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