CONCEPTS AS OBJECTS

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1. Concepts (including propositions) as objects

2. Functions from objects to concepts of them.

3. Concepts and propositions are not a natural kind.

    - There are a variety of useful spaces of concepts.

    - Concepts are (usually) approximate entities.

Concepts and propositions—1

"...it seems that hardly anybody proposes to use different variables for propositions and for truth-values, or different variables for individuals and individual concepts."—(Carnap 1956, p. 113).

Variables for propositions and individuals are written in lower case, e.g. p and x. Variables for propositions and individual concepts are capitalized, e.g. P and X.

This talk is about expressiveness rather than for presenting a theory.

Concepts and propositions–2

We write

$denotes(Mike, mike)$ or when functional, $mike = denot(Mike)$

$Telephone(Mike)$ is the concept of Mike's telephone number

$denot(Telephone(Mike)) = telephone(mike)$

Knowing what and knowing that

$knows(pat, Telephone(Mike))$

Suppose $telephone(mike) = telephone(mary)$

$Telephone(Mike) \neq Telephone(Mary)$

Possibly, $\neg knows(pat, Telephone(Mary))$

Truth values and propositions:
$man(mike)$
$true(Man(Mike)$
$knows(pat, Man(Mike))$ means Pat knows whether Mike is a man.
Possibly $knows(pat, Man(Mike)) \wedge \neg man(mike)$

$k(pat, Man(Mike)) \equiv true(Man(Mike)) \wedge knows(pat, Man(Mike))$

Equality and Existence

$true(Telephone(Mike)\ EqualsC\ Telephone(Mary)$, although
$Telephone(Mike) \neq Telephone(Mary)$
$telephone(denot(Mike)) = telephone(denot(Mary))$
$telephone(mike) = telephone(mary)$
$denot(Telephone(Mike)) = denot(Telephone(Mary))$

$$(\forall X)(exists(X) \equiv (\exists x)denotes(X, x))$$

$ishorseCPegasus$
$Winged(Pegasus)$
?$true(\text{Winged-Horse}(Pegasus))$
$true(\text{Greek mythology}, \text{Winged-Horse}(Pegasus))$
$\neg exists(Pegasus)$

We can have

$$(\exists X)(exists(\text{Greek Mythology}, X) \wedge \text{Winged-Horse}(X)),$$

but most likely, there doesn't have to be a domain of Greek mythological objects. This suggests that some of the rules of inference of predicate logic be weakened for such theories.

About propositions

$$true(Not(P)) \equiv \neg true(P)$$
$$true(P\ And\ Q) \equiv true(P) \wedge true(Q)$$
? $P\ And\ Q = Q\ And\ P$
? $P\ And\ (Q\ Or\ R) = (P\ And\ Q)\ Or\ (P\ And\ R)$

This way lies NP-completeness and even undecidablity of whether two formulas name the same proposition.

Functions from things to concepts

Numbers can have standard concepts $Concept1(n)$ is a certain standard concept of the number n. Writing $Concept1$ suggests that there might be another mapping $Concept2$ from numbers to concepts of them.

We can have

$\neg knew(kepler, CompositeC(Number(Planets)))$,
and also
$knew(kepler, (CompositeC(Concept1(denot(Number(Planets))))))$.

Functions from things to concepts–2

Russell's example: *I thought your yacht was longer thann it is.* can be treated similarly, although it requires a function going from the concept $Length(Youryacht)$ to what I thought its value was.

$$denot(I, Length(Youryacht)) > length(youryacht)$$

Functions from things to concepts–3

We may also want a map from things to concepts of them in order to formalize a sentence like, *"Lassie knows the location of all her puppies"*. We write this

$$(\forall x)(ispuppy(x, lassie) \supset knowsd(lassie, LocationdC(Conceptd(x)))).$$

$Conceptd$ takes a puppy into a dog's concept of it, and $Locationd$ takes a dog's concept of a puppy into a dog's concept of its location. The axioms satisfied by $knowsd$, $Locationd$ and $Conceptd$ can be tailored to our ideas of what dogs know.

$$\begin{aligned}&(\exists n2)(k(pat, Concept2(n2)\ EqualsC\ Telephone(Mike)))\\&\qquad \equiv knows(pat, Telephone(Mike))\end{aligned}$$

or

$$\begin{aligned}&knows(pat, Telephone(Mike))\\&\qquad \equiv denot(pat, Telephone(Mike)) = telephone(mike).\end{aligned}$$

Concepts as approximate entities

• Approximate entities occur in human common sense reasoning. They don't have if-and-only-if definitions, e.g. the rock and ice constituting Mount Everest.

• The set of individual concepts of Greek mythology is another approximate entity. Few of them have denotations.

• The logical way of handling approximate entities is to axiomatize them weakly. Did Pegasus have a mother?

• $exists(\text{Greek Mythology}, Pegasus)$,
$\neg exists(\text{Greek Mythology}, Thor)$,
$\neg exists(\text{Greek Mythology}, \text{George Bush})$,
$exists(\text{Greek Mythology}, Mother(Pegasus))$?

Mr. S and Mr. P

Two numbers m and n are chosen such that $2 \leq m \leq n \leq 99$. Mr. S is told their sum and Mr. P is told their product. The following dialogue ensues:

Mr. P: I don't know the numbers.
Mr. S: I knew you didn't know. I don't know either.
Mr. P: Now I know the numbers.
Mr S: Now I know them too.

In view of the dialogue, what are the numbers?

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Formalizing Mr. S and Mr. P

$knows(person, pair, time), k(person, Proposition, time)$
persons: s, p, sp
$\neg knows(p, Pair0, 0)$
$knows(s, Sum(Pair0), 0)$
$knows(p, Product(Pair0), 0)$
$(\forall pair)(sum(pair) = sum(pair0)$
$\quad \rightarrow \neg k(s, Not(Pair0\ Equal\ Concept1(pair)), 0))$
$k(sp, \ldots, 0)$

In the paper $A(w1, w2, person, time)$ means that in world $w1$, world $w2$ is possible for $person$ at $time$.

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