Graphs and colorings for Answer Set Programming with Preferences: Extended Abstract[a]

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- Motivation (Problem Description)
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[a] This is an extended abstract of (Konczak, Schaub, & Linke 2003)

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Motivation

The notion of *preference* in commonsense reasoning is pervasive.

For instance,

- in buying a car, one may prefer certain features over others;
- in scheduling, meeting some deadlines may be more important than meeting others;
- in legal reasoning, laws are subject to higher principles, like lex superior or lex posterior, which are themselves subject to "higher higher" principles;
- etc etc . . .

Problem Description

- **Goal:** integrate preference information in an answer set solver
- **Requirements:**
  - Underlying program:
    - ordered logic program $(\Pi, <)$
    - $<$ is a strict *partial order* among rules
    - $<$ selects preferred answer sets from standard ones of Π
  - Underlying semantics: (Delgrande, Schaub, Tompits 2003)
- **Solution:**
  - including preference information in the *rule dependency graph*
  - defining order preserving graph *colorings* representing preferred answer sets

Underlying semantics of preferred answer sets

Let $(\Pi, <)$ be an ordered program and let X be an answer set of Π. Then, X is called $<$-*preserving*, if there exists an enumeration $\langle r_i \rangle_{i \in I}$ of $R_\Pi(X)$ [a] such that for every $i, j \in I$ we have that:

1. if $r_i < r_j$, then $j < i$,
2. $body^+(r_i) \subseteq \{head(r_j) \mid j < i\}$, and
3. if $r_i < r'$ and $r' \in \Pi \setminus R_\Pi(X)$, then
   (a) $body^+(r') \not\subseteq X$ or
   (b) $body^-(r') \cap \{head(r_j) \mid j < i\} \neq \emptyset$.

[a] $R_\Pi(X) = \{r \in \Pi \mid body^+(r) \subseteq X \text{ and } body^-(r) \cap X = \emptyset\}$

Solution: Rule Dependency Graph

Let $(\Pi, <)$ be an ordered logic program.

The **ordered rule dependency graph** (*DG*) $\Gamma_{(\Pi,<)} = (\Pi, E_0, E_1, E_2)$ of $(\Pi, <)$ is a labeled directed graph with

$$
\begin{aligned}
E_0 &= \{(r, r') \mid r, r' \in \Pi, head(r) \in body^+(r')\} && (0-edges);\\
E_1 &= \{(r, r') \mid r, r' \in \Pi, head(r) \in body^-(r')\} && (1-edges);\\
E_2 &= \{(r, r') \mid r, r' \in \Pi, r' < r\} && (2-edges).
\end{aligned}
$$

This definition extends the one in (Linke,01) by 2-edges for representing preferences among rules.

Solution: Colorings

Given an ordered logic program $(\Pi, <)$ along with its *DG* $\Gamma_{(\Pi,<)}$.

- C is a **(partial) coloring** of $\Gamma_{(\Pi,<)}$ if C is a (partial) mapping $C : \Pi \to \{\oplus, \ominus, \oslash\}$
- C is total if $C_\oplus \cup C_\ominus = \Pi$ and $C_\oslash = \emptyset$
- color $\oplus$ indicates *application* of a rule
- color $\ominus$ indicates *non-application* of a rule
- color $\oslash$ is an intermediate version of $\ominus$. [a]
- color vertex r with $\oslash$, if r must be colored with $\ominus$ but there is not yet any justification for coloring r with $\ominus$.

[a] The color $\oslash$ is comparable to dlv's "`must_be_true`".

Solution: Order preserving admissible colorings

For every preferred answer set X of $(\Pi, <)$, define a unique $<$-**preserving admissible coloring** C of $\Gamma_{(\Pi,<)}$ as

$$C = (R_\Pi(X), \Pi \setminus R_\Pi(X), \emptyset)$$

where

$$R_\Pi(X) = \{r \in \Pi \mid body^+(r) \subseteq X \text{ and } body^-(r) \cap X = \emptyset\} \ .$$

To find order preserving admissible colorings, we look for a sequence of operations turning the uncolored graph into a totally colored one. [a]

[a] This concept of a derivation is inspired by an SLD-derivation.

Solution: Operators

The following operators extend partial colorings, where every operator avoids supporting (and blocking of) higher ranked rules by lower ranked ones (wrt $<$).

- *Deterministic operators*:
  - $\mathcal{P}$: basic forward propagation along the preference order $<$
  - $\mathcal{U}$: allow for detecting unsupported rules
  - $(\mathcal{PU})^*$ computes the closure under $\mathcal{P}$ and $\mathcal{U}$
- *Nondeterministic operators*: $\mathcal{D}^\circ$ for $\circ \in \{\oplus, \oslash\}$:
  - respects the order $<$
  - choose a rule and color it, either with $\oplus$ or $\oslash$
  - $\mathcal{D}^\oplus$ is restricted to choosing supported rules only
  - $\mathcal{D}^\oslash$ is restricted to choosing not supported rules only

Solution: Operational characterization

Let $\Gamma_{(\Pi,<)}$ be the *DG* of ordered logic program $(\Pi,<)$ and let C be a total coloring of $\Gamma_{(\Pi,<)}$.

Then, C is a $<$- preserving admissible coloring of $\Gamma_{(\Pi,<)}$ iff there exists a sequence $(C^i)_{0\leq i\leq n}$ with the following properties:

1. $C^0 = (\mathcal{PU})^*_{\Gamma_{(\Pi,<)}}((\emptyset,\emptyset,\emptyset))$;
2. $C^{i+1} = (\mathcal{PU})^*_{\Gamma_{(\Pi,<)}}(\mathcal{D}^\circ_{\Gamma_{(\Pi,<)}}(C^i))$ for some $\circ \in \{\oplus, \oslash\}$ and $0 \leq i < n$;
3. $C^n = C$.

Solution: Example I

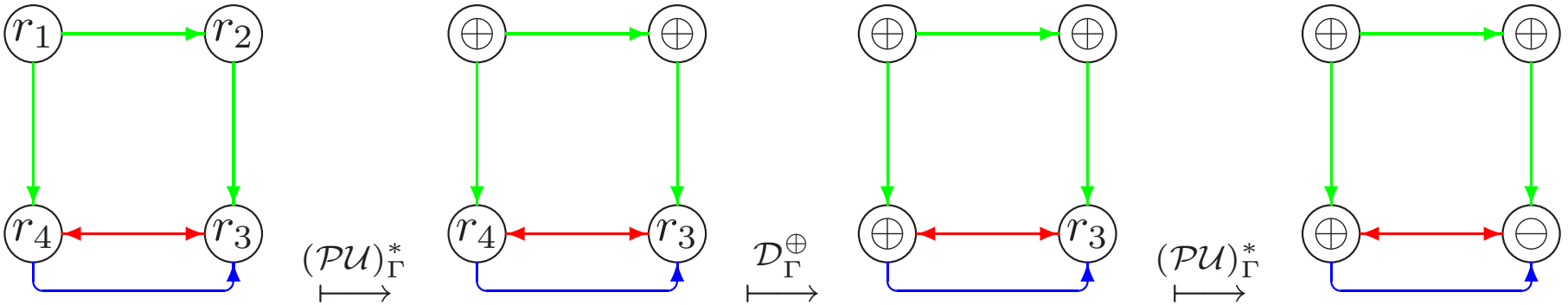
$r_1: \quad p \leftarrow$

$r_2: \quad b \leftarrow p$

$r_3: \quad f \leftarrow b, not\ f'$

$r_4: \quad f' \leftarrow p, not\ f$

Preferences: $r_3 < r_4$



Solution: Example II

$r_1: \quad a \leftarrow c$

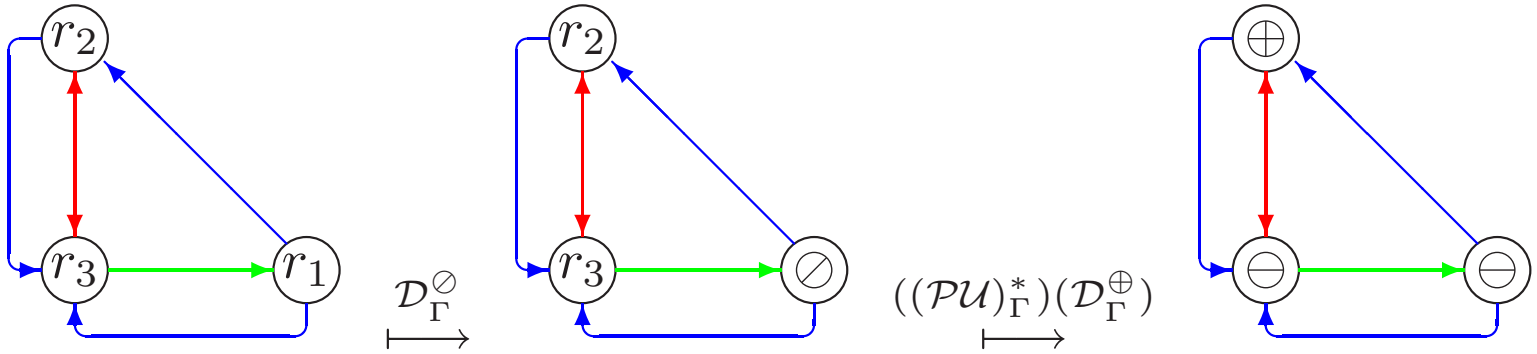
$r_2: \quad b \leftarrow not\ c$

$r_3: \quad c \leftarrow not\ b$

Preferences:

$r_2 < r_1$

$r_3 < r_2$



Further Thesis Work

- Well-founded semantics with preferences
- Abduction of preferences
- Voting theory
- Applications:
  - Include preferences in scheduling a meeting
  - Abduction and preferences adapted to linguistic problems

Related work

Preferred answer sets:

- `plp` system as a pre-compilation front-end
- (Eiter et al. 2003) meta-interpreter
- (Konczak, Schaub, Linke 2003)

Standard answer sets:

- (Konczak, Linke, Schaub 2003)
- (Linke, 01)

References

(Delgrande, Schaub, Tompits 2003) A framework for compiling preferences in logic programs. TPLP 2003 (*)

(Eiter et al. 2003) Computing preferred answer sets by meta-interpretation in answer set programming, TPLP 2003

(Konczak, Schaub, Linke 2003) Graphs and colorings for answer set programming with preferences. Fundamenta Informaticae, volume 57, number 2-4, 2003 (*)

(Konczak, Linke, Schaub 2003) Graphs and colorings for answer set programming: Abridged report (LPNMR'04) (*)

(Linke, 01) T. Linke, Graph theoretical characterization and computation of answer sets, IJCAI 01 (*)

(PLP) http://www.cs.uni-potsdam.de/~torsten/plp/

(*) available at http://www.cs.uni-potsdam.de/wv/publications/