CS4725: Solutions to Homework 1

Part I Translating English to first-order logic

- Variables: p ranges over people; v ranges over volumes; b ranges over books
- Constants: Sam, Barbara, Tolstoy, Joyce (all people)
- Predicates:
  - Owns(p, v) : Person p owns volume v
  - Author(p, b): Person p wrote book b
  - Copy(v, b): Volume v is a copy of book b.

Express each of the following as a sentence in L:

[1] Sam owns a copy of every book that is either by Tolstoy or Joyce.

$$(\forall b)(Author(Tolstoy, b) \vee Author(Joyce, b)) \Rightarrow \exists v(Copy(v, b) \wedge Owns(Sam, v))$$

[2] All the volumes that Barbara owns are copies of books by Tolstoy.

$$\forall v Owns(Barbara, v) \Rightarrow \exists b(Copy(v, b) \wedge Author(Tolstoy, b))$$

[3] If Barbara owns a copy of a book, then Sam owns a copy of the same book.

$$\forall b, v(Owns(Barbara, v) \wedge Copy(v, b)) \Rightarrow \exists v1(Owns(Sam, v1) \wedge Copy(v1, b))$$

[4] There is some book that Sam owns but Barbara doesn't.

$$\exists b, v1(Owns(Sam, v1) \wedge Copy(v1, b) \wedge \forall v2(copy(v2, b) \Rightarrow \neg(Owns(Barbara, v2)))$$

[5] Every author owns a copy of each of his own books.

$$\forall x, y(Author(x, y) \Rightarrow \exists z(Copy(z, y) \wedge Owns(x, z)))$$

Variables and predicates for 6–10:

- Variables: x, y range over people. z, w range over grades. c ranges over courses. l ranges over languages. o ranges over objects.
- $Vegetarian(x)$ x is a vegetarian
- $Acquainted(x, y)$ x is acquainted with y
- $Score(w, c)$ w is a score in course c
- $HistoryCourse(c)$ c is a history course
- $BiologyCourse(c)$ c is a biology course
- $Greater(z, w)$ z is greater than w
- $Owns(x, o)$ x owns object o
- $BreadMachine(o)$ o is a bread machine
- $AmateurBaker(x)$ x is an amateur baker
- $Speaks(x, l)$ x speaks language l
- $Swiss(x)$ x is Swiss

[6] Only one person is a vegetarian.

$$\exists x(Vegetarian(x) \wedge \forall y(Vegetarian(y) \Rightarrow x = y))$$

[7] No person is acquainted with all vegetarians.

$$\neg\exists x\forall y(Vegetarian(y) \Rightarrow Acquainted(x, y))$$

[8] The best score in History is better than the best score in Biology.
$\forall z1, z2, w1, w2, c1, c2$ *HistoryCourse(c1)* $\wedge$ *BiologyCourse(c2)* $\wedge$
Score(z1,c1) $\wedge$ *Score(z2,c1)* $\Rightarrow$ *Greater(z1,z2)* $\wedge$ *Score(w1,c2)* $\wedge$ *Score(w2,c2)* $\Rightarrow$ *Greater(w1,w2)*
$\Rightarrow$ *Greater(z1,w1)*

[9] Nobody except amateur bakers owns bread machines.

$$\forall x, o(Owns(x, o) \wedge BreadMachine(x) \Rightarrow AmateurBaker(x))$$

[10] All Swiss speak the same languages.

$$\forall x, y, l(Swiss(x) \wedge Swiss(y) \wedge Speaks(x, l) \Rightarrow Speaks(y, l))$$

[11] **Definition of grandparent:**

$$\forall x, y(grandparent(x, y) \iff (\exists w(parent(x, w) \wedge parent(w, y))))$$

[12] **Definition of sister-in-law.**
Note that we are considering three cases (sufficient conditions) here. x is a sister-in-law of y if x is married to someone who is y's brother (e.g., Diana was Prince Andrew's sister-in-law); or, if x is a sister of someone who is married to y (e.g., Princess Anne, Prince Charles's sister, was Diana's sister-in-law), or if x is married to someone whose sibling is married to y (e.g., Diana was Sara Ferguson's sister-in-law, and this relationship was reciprocal; in addition, Diana was the sister-in-law of Anne's current husband, Captain Timothy Laurence). In all cases x must be female. Note also that the third case is not always considered when defining sister-in-law, so no credit was deducted if this case was not considered.

$Sister\text{-}in\text{-}law(x,y) \iff$
$\quad Female(x) \wedge$
$\quad ((\exists z\ Married(x,z) \wedge Brother(z,y)) \vee$
$\quad ((\exists z\ Sister(x,z) \wedge Married(z,y)) \vee$
$\quad ((\exists w, z\ Married(x,z) \wedge Sibling(z,w) \wedge Married(w,y)))$

[13] **Definition of first-cousin(x, y):**

It would probably have been simpler to define first cousins as people having parents who are siblings, and then to prove as a theorem that first cousins have grandparents in common. But the instructions were to define first-cousinhood starting off with the characterization that first cousins have grandparents in common. A plausible initial effort would have been to take having grandparents in common as the sufficient and necessary condition for (i.e., the definition of) being first cousins. However, some reflection, as well as working through the problems in part III, shows that this will not work. First, two siblings have grandparents in common even though they are not first cousins. Second, since you are eventually required to prove that the parents of first cousins are siblings, and the defintion of sibling refers to two parents, considering *one* grandparent will not be sufficient. In fact, you need to consider 4 individuals, the 2 grandparents shared by the cousins, and a parent for each cousin.
Thus, the definition is:

$\forall x, y\ (Cousin(x,y) \iff$
$(\exists g1, g2, p1, p2\ (mother(g1,p1) \wedge mother(g1,p2) \wedge father(g2,p1) \wedge father(g2,p2) \wedge parent(p1,x)$
$\wedge\ parent(p2,x) \wedge x \neq y)$

[14] **Define** mth **cousin** n **times removed.** This is done in 2 steps, first defining the concept of mth cousin, and then defining the concept of mth cousin n times removed.

mth cousin:

$\forall x, y$ *MCousin(1,x,y)* $\iff$ *cousin(x,y)*

$\forall x, y, m_{\geq 2}$ *MCousin(m,x,y)* $\iff \exists z, w$ *Parent(z,x)* $\wedge$ *Parent(w,y)* $\wedge$ *MCousin(m-1,z,w)*

$\forall x, y, m$ *MCousinRemove(m,1,x,y)* $\iff \exists w Parent(w, y) \wedge MCousin(m, x, w)$

$\forall x, y, m, n_{\geq 2} MCousinRemove(m, n, x, y) \iff \exists w Parent(w, y) \wedge MCousinRemove(m, n - 1, x, w)$

Part II Translating first-order logic to English

[1] $(\forall x) Hesitates(x) \Rightarrow Lost(x)$
He who hesitates is lost

[2] $\neg(\exists x) Business(x) \wedge Like(x, Showbusiness)$
There's no business like show business

[3] $\neg(\forall x) Glitters(x) \Rightarrow Gold(x)$
All that glitters is not gold. (This is the well-known adage).

[4] $(\forall x) Glitters(x) \Rightarrow \neg Gold(x)$
Whatever glitters is not gold.

[5] $(\exists x)(\forall y) Loves(x, y)$
There is someone who loves everyone. (True, for example, where x and y range over people, and x is taken to be Mother Teresa)

[6] $(\forall y)(\exists x) Loves(x, y)$
Everyone is loved by someone (True, for example, in a non-dysfunctional world where everybody loves his or her children; then every person is loved by his or her mother.)

[7] $(\exists y)(\forall x) Loves(x, y)$
Someone is loved by everyone. (For example, Joe DiMaggio was said to be universally beloved.)

[8] $(\forall x)(\exists y) Loves(x, y)$
Everybody loves someone. (True in a non-dysfunctional world where everyone loves his or her parents.)

[9] $((\exists x) P(x)) \iff Q(x)$
Interpretation that makes this true: Let P be a one-place predicate meaning has composed *The Magic Flute*; let Q be a one-place predicate meaning has lived. Let the second occurrence of x (the unbound one) represent a constant, namely Mozart. Then the sentence says that

there is someone who has composed *The Magic Flute* if and only if Mozart has lived, which is true (unless we assume that someone else would have composed *The Magic Flute* had Mozart not lived).

[10] $(\exists x)(P(x) \Longleftrightarrow Q(x))$
Interpretation that makes this true: Let x range over planets; let P(x) be true if x is the first planet to be seen in the evening; let Q(x) be true if x is the last planet to be seen in the morning. The sentence is true as long as there is some planet that is first to be seen in the evening if and only if it is last to be seen in the evening. (Indeed, all planets in the solar system work here: P(Venus) and Q(Venus) are both true; and for all other planets, P(x) and Q(x) are both false.)

Part III Inference in first–order logic (natural deduction)

Note that there are many ways to write down the assumptions of the family tree. If you write down parent(Elizabeth, Charles), parent(Philip, Charles), etc., that makes it easier for problem 1, but harder for problem 2. If you write down mother(Elizabeth, Charles), father(Philip, Charles), etc., that makes it harder for problem 1, easier for problem 2.

1. Prove that Philip is William's grandfather.

Here's one solution:

```
{1}     1. Father(Philip, Charles)            Premise
{2}     2. Father(Charles, William)           Premise
{3}     3. forall x,y
           (Father(x,y) <==> Parent(x,y) & Male(x))  Premise  (def.)
{4}     4. forall x,y
           (Grandparent(x,y) <==>
               exists z (Parent(x,z) & Parent(z,y))) Premise (def.)
{5}     5. forall x,y
           (Grandfather(x,y) <==> Grandparent(x,y) & Male(x)) Premise (def.)
{3}     6. forall y
            (Father(Philip,y) <==> Parent(Philip,y) & Male(Philip)) U.S., 3
{3}     7. Father(Philip,Charles) <==> Parent(Philip,Charles) & Male(Philip)
                                                                 U.S., 6
{3}     8. forall y
            (Father(Charles,y) <==> Parent(Charles,y) & Male(Charles)) U.S., 3
{3}     9. Father(Charles, William)
                <==> Parent(Charles,William) & Male(Charles)
```

```
                                                               U.S., 8
{1,3}  10. Parent(Philip,Charles)                              T 1,7
{1,3}  11. Male(Philip)                                        T 1,7
{2,3}  12. Parent(Charles,William)                             T 2,9
{1,2,3}13. Parent(Philip,Charles) & Parent(Charles, William)   T 10.12
{1,2,3}14. exists z
             (Parent(Philip,z) & Parent(z,William))            E.G., 13
{4}    15. forall y
             (Grandparent(Philip,y) <==>
               exists z (Parent(Philip,z) & Parent(z,y)))      U.S., 4
{4}    16. Grandparent(Philip,William) <==>
               exists z (Parent(Philip,z) & Parent(z,William)) U.S., 15
{1,2,3,4} 17. Grandparent(Philip,William)                      T 14,16
{5}       18. forall y
                (Grandfather(Philip,y) <==> Grandparent(Philip,y) & Male(Philip))
                                                               U.S. 5
{5}       19. Grandfather(Philip,William) <==>
                Grandparent(Philip,William) & Male(Philip)    U.S. 18
{1,2,3,4,5}  20.  Grandfather(Philip, William)                T 11,17, 19
```

2. Prove that Beatrice and Harry are first cousins.

How this proof is done depends on how the concept of first cousin is defined. If first cousins are defined as individuals who share a grandparent, the definition of sibling previously provided would have to be weakened in order to prove problem 3. (Specifically, one would have to redefine siblings as individuals who share at least one parent rather than both a father and a mother.) Alternately, one can provide a stronger definition of cousin, which is the route taken below. Note that there are existential quantifiers over 4 individuals, representing the two grandparents shared by the cousins as well as a parent of each cousin. Note also that to be strictly accurate, one must state that the two parents of the cousins are distinct; otherwise, the cousins would themselves be siblings.

```
{1}     1. Father(Philip, Charles)                             Premise
{2}     2. Father(Philip, Andrew)                              Premise
{3}     3. Mother(Elizabeth, Charles)                          Premise
{4}     4. Mother(Elizabeth, Andrew)                           Premise
{5}     5. Father(Charles, Harry)                              Premise
{6}     6. Father(Andrew, Beatrice)                            Premise
{7}     7. ~(Charles=Andrew)                                   Premise
{8}     8.  forall x, y
```

```
            (Father(x,y) <==> Parent(x,y) & Male(x,y))         Premise (def)
{9}     9. forall x,y
           (Cousin(x,y) <==>
            exists g1,g2,p1,p2
             (mother(g1,p1) & mother(g1,p2) &
              father(g2,p1) & father(g2,p2) &
              parent(p1,x)  & parent(p2,x) & ~(x=y)))          Premise (def)
{8}    10. forall y
           Father(Charles,y) <==> Parent(Charles,y) & Male(Charles) U.S. 8
{8}    11. Father(Charles, Harry) <==> Parent(Charles,Harry) & Male(Charles)
                                                                 U.S. 10
{5,8}  12. Parent(Charles, Harry)                              T 5,11
{8}    13. forall y
           Father(Andrew, y) <==> Parent(Andrew, y) & Male(Andrew)  U.S. 8
{8}    14. Father(Andrew, Beatrice)
                <==> Parent(Andrew, Beatrice) & Male(Andrew)
                                                                 U.S. 13
{6,8}  15. Parent(Andrew, Beatrice)
{1,2,3,4,5,6,7,8}
       16.  Mother(Elizabeth, Charles) & Mother(Elizabeth, Andrew) &
            Father(Philip, Charles) & Father(Philip, Andrew) &
            Parent(Charles, Harry) & Parent(Andrew, Beatrice)
                                 & ~(Charles=Andrew)
                                                        T 1,2,3,4,7,12,15
{1,2,3,4,5,6,7,8,}
       17. exists p2
           Mother(Elizabeth,Charles) & Mother(Elizabeth,p2) &
           Father(Philip, Charles) & Father(Philip, p2) &
           Parent(Charles, Harry) & Parent(p2,Beatrice)
                               &~(Charles=p2)            E.G. 16
{1,2,3,4,5,6,7,8}
       18. exists p1,p2
           Mother(Elizabeth,p1) & Mother(Elizabeth,p2) &
           Father(Philip, p1) & Father(Philip,p2) &
           Parent(p1,Harry) & Parent(p2,Beatrice) & ~(p1=p2)  E.G. 17
{1,2,3,4,5,6,7,8,}
       19. exists g2,p1,p2
           Mother(Elizabeth,p1) & Mother(Elizabeth,p2) &
           Father(g2,p1) & Father(g2,p2) &
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```
          Parent(p1,Harry) & Parent(p2,Beatrice) & ~(p1=p2)  E.G. 18
{1,2,3,4,5,6,7,8}
      20. exists g1,g2,p1,p2
          Mother(g1,p1) & Mother(g1,p2) & Father(g2,p1)
          & Father(g2,p2) & Parent(p1,Harry) &
          Parent(p2,Beatrice) & ~(p1=p2)                     E.G. 19
{9}   21. Cousin(Beatrice, Harry) <==>
          exists g1,g2,p1,p2
          Mother(g1,p1) & Mother(g1,p2) & Father(g2,p1)
          & Father(g2,p2) & Parent(p1,Harry) &
          Parent(p2,Beatrice) & ~(p1=p2)                     U.S. (twice),9
{1,2,3,4,5,6,7,8,9}
      22. Cousin(Beatrice, Harry)                            T 20,21
```

3. Prove that first cousins have parents who are siblings

```
{1}   1. Def. of cousin, from line 9 above                    Premise (def)
{2}   2. Def. of sibling, from the homework                   Premise (def)
{3}   3. Cousin(A,B)                                          Premise
{3}   4. Cousin(A,B) <==>
         exists g1,g1,p1,p2
         Mother(g1,p1) & Mother(g1,p2) & Father(g2,p1) &
         Father(g2,p2) & Parent(p1,A) & Parent(p2,B) & ~(p1=p2) U.S. (2x), 1
{1,3} 5. exists p1,p2, g1,g2
       Mother(g1,p1) & Mother(g1,p2) & Father(g2,p1) &
       Father(g2,p2) & Parent(p1,A) & Parent(p2,B) & ~(p1=p2) T 1,4

{6}   6. exists g1,g2
         Mother(g1,P1) & Mother(g1,P2) & Father(g2,P1) &
         Father(g2,P2) & Parent(P1,A) & Parent(P2,B) & ~(P1=P2) E.S. Premise,5
        [it would be best to apply the E.S. premise rule 2 times,
          and then discharge 2 times at the end]
{2}   7. Sibling(P1,P2) <==>
          exists g1,g2
           Mother(g1,P1) & Mother(g1,P2) & Father(g2,P1) &
           Father(g2,P2) & Parent(P1,A) & Parent(P2,b) & ~(P1=P2) U.S. (2x), 2
{2,6} 8. Parent(P1,A) & Parent(P2,B) & Sibling(P1,P2)     T 6,7
{2,6} 9. exists p1,p2
          Parent(p1,A) & Parent(p2,B) & Sibling(p1,p2)    E.G., 8
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{1,2,3}10. exists p1,p2
            Parent(p1,A) & Parent(p2,B) & Sibling(p1,p2)
                                                            E..S. Discharge 5,6,9
{1,2} 11. Cousin(A,B) ==>
           exists p1,p2
            Parent(p1,A) & Parent(p2,b)  & Sibling(p1,p2)  Cond. Discharge 3,10
{1,2} 12.  forall x,y
           Cousin(x,y) ==>
             Parent(p1,x) & Parent(p2,x) & Sibling(p1,p2)  U.G. (2x), 11
```

4. From (forall x) (P(x) & Q(x)) prove (forall x)(P(x)) & (forall x) (Q(x)

```
From (forall x) (P(x) & Q(x)) prove (forall x)(P(x)) & (forall x) (Q(x)

{1}  1.  (forall x) (P(x) & Q(x))                           Premise
{1}  2.  P(A) & Q(A)                                        U.S. 1
{1}  3.  P(A)                                               T  2
{1}  4.  (forall x) (P(x))                                  U.G. 3
{1}  5.  Q(A)                                               T 2
{1}  6.  (forall x) (Q(x))                                  U.G. 5
{1}  7.  (forall x) (P(x)) & (forall x) (Q(x))              T 4,6

The converse does hold.  The proof follows below (but is not required
for credit):

{1} 1.   (forall x) (P(x)) & (forall x) (Q(x))              Premise
{1} 2.   (forall x) (P(x))                                  T 1
{1} 3.   P(A)                                               U.S. 2
{1} 4.   (forall x) (Q(x))                                  T 1
{1} 5.   Q(A)                                               U.S. 4
{1} 6.   P(A) & Q(A)                                        T 3,5
{1} 7.   (forall x) (P(x) & Q(x))                           U.G. 6
```

5. From (forall x)(P(x)) v (forall x)(Q(x)) Prove (forall x)(P(x) v Q(x))

```
From (forall x)(P(x)) v (forall x)(Q(x)) Prove (forall x)(P(x) v Q(x))

The strategy here is to prove (as a theorem, i.e. from the empty
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set of assumptons):
(forall x)(P(x)) v (forall x)(Q(x))  ==>
(forall x)(P(x) v Q(x))
```

```
By the deduction theorem, this is equivalent to proving that the
second statement follows from the first.
```

```
{1} 1. (forall x) (P(x))                                    Premise
{1} 2. P(A)                                                 U.S. 1
{1} 3. P(A) v Q(A)                                          T 2
{1} 4. (forall x) (P(x) v  Q(x))                            U.G. 3
{ } 5. (forall x) (P(x) ==> (forall x) (P(x) v  Q(x))       Cond. Discharge 1,4
{ } 6. (forall x) (P(x)) v (forall x) (Q(x)) ==>            T 5
          (forall x) (P(x) v Q(X))
```

```
The converse does not hold.  For example, if x ranges over
all U.S. senators, P(x) is true if x is a Democrat, and Q(x) is
true if x is a Republican, then it is true that any U.S. senator
is either a Democrat or a Republican, but it is certainly not true
that either all senators are Democrats or all are Republicans.
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6: From $(\forall x)(P(x) \vee Q(x)), (\exists x)(\neg Q(x)), (\forall x)(S(x) \Rightarrow \neg P(x)), prove (\exists x)(\neg S(x)))$

```
{1}       1. (forall x) (P(x) v Q(x))                         Premise
{2}       2. (exists x) ~ Q(x)                                Premise
{3}       3. (forall x) (S(x) ==> ~ P(x))                     Premise
{4}       4. ~Q(A)                                            E.S. Premise,2
{1}       5. P(A) v Q(A)                                      U.S. 1
{1}       6. ~Q(A) ==> P(A)                                   T 5
{3}       7. S(A) ==> ~P(A)                                   U.S. 3
{3}       8. P(A) ==> ~S(A)                                   T 7
{1,3,4}  9. ~S(A)                                             T 4,6,8
{1,3,4} 10. (exists x) ~ S(X)                                 E.G. 9
{1,2,3} 11. (exists x) ~ S(X)                                 E.S. Discharge 2,4,10
```

7: Proving that Sam owns some book that Barbara does not own.

Overview: The best way to approach this problem is to look at the starting assumptions and try to get to the intermediate points in the argument. It beomes apparent that there are some gaps as one tries to prove the intermediate points from the original assumptions. One then adds the assumptions needed to make the proof valid.

We start out with 2 assumptions, 1. and 2. of Part I (i. and ii. of 3. on Davis, p. 92).

Sam owns a copy of every book that is either by Tolstoy or Joyce.

$$(\forall b)(Author(Tolstoy, b) \vee Author(Joyce, b)) \Rightarrow \exists v(Copy(v, b) \wedge Owns(Sam, v))$$

All the volumes that Barbara owns are copies of books by Tolstoy.

$$\forall v Owns(Barbara, v) \Rightarrow \exists b(Copy(v, b) \wedge Author(Tolstoy, b))$$

From 1., we are supposed to get to intermediate step (I): Sam owns all books by Joyce:

$$(I)\forall b(Author(Joyce, b) \Rightarrow \exists v(Copy(v, b) \wedge Owns(Sam, v))).$$

It is clear that (I) follows directly from 1., with no further assumptions. This is done below in steps 7. - 11. of the proof.

From 2., we are supposed to get to intermediate step (II): Barbara owns only books by Tolstoy:

$$(II)\forall b, v Owns(Barbara, v) \wedge Copy(v, b) \Rightarrow Author(Tolstoy, b)$$

.

(II) does *not* follow directly from 2.. In particular, suppose that a volume can be a copy of 2 different books. Then all of Barbara's volumes may be copies of books by Tolstoy, but conceivably, one volume may also at the same time be a copy of Joyce. To preclude this possibility, we add the assumption that any volume is a copy of only one book:

$$\forall v, b1, b2 Copy(v, b1) \wedge Copy(v, b2) \Rightarrow b1 = b2$$

The proof of (II) from 2. and this extra assumption take place in steps 12. – 21. of the proof. It should be noted that step 18. uses as justification the principle of substitution of equals. Actually, this principle isn't one of the natural deduction rules given in Davis, Chapter 1, or Mates, Chapter 7. That is because Davis, Chapter 1 and Mates, Chapter 7 give the rules of natural deduction for first-order logic without equality. This problem requires using a first-order logic with equality, and thus we need the extra inference rule that allows substitution of equals. (Alternately, we could use the axioms of equality, given in Davis, p. 43 (Table 2.4). Or, we could add yet another assumption, such as

$$\forall a, b1, b2 author(a, b2) \wedge b1 = b2 \Rightarrow author(a, b1)$$

Given that this part of the proof was problematic, particularly since you weren't provided with the substitution of equals inference rule, it was also okay to just take II as an assumption.)

From (I) and (II), we are supposed to get to intermediate step (III): Any book by Joyce is owned by Sam but not by Barbara:

$$\forall b(Author(Joyce, b) \Rightarrow \exists v(Copy(v, b) \wedge Owns(Sam, v) \wedge \forall v1 \neg(Copy(v1, b) \wedge Owns(Barbara, v1))$$

It will obviously be easy to prove that any book by Joyce is owned by Sam, because that was proved above. To show that any book by Joyce is not owned by Barbara, we will have to add two assumptions:
Joyce didn't co-author any books (to preclude the possibility that Joyce co-authored a book with Tolstoy; Barbara might in such a case indeed own a book by Joyce):

$$\forall b, p(Author(Joyce, b) \wedge p \neq Joyce \Rightarrow \neg Author(p, b))$$

and
Joyce and Tolstoy are distinct people:

$$Joyce \neq Tolstoy$$

We can then prove III. (Steps 22.-30. of the proof.)

Finally, we must go from intermediate step (III) to the fact that Sam owns some book that Barbara doesn't own. To do this, we will have to add the step that Joyce wrote at least one book. (If Joyce never wrote a book, then Sam could vacuously own all of Joyce's books, and in fact, he might not own a book that Barbara didn't own):

$$\exists b Author(Joyce, b)$$

With this assumption, the proof is completed. (Steps 31.-34.)

Note that the proof below is somewhat longer than it needs to be. In particular, since I am going through each of the intermediate steps, and trying to give a complete proof for each mini-step, some proof steps are repeated. (See, for example, how the premise Author(J,B) is introduced 3 times in the proof: at step 8., step 25., and step 31.; introducing it once would have been enough.

The proof follows below:

```
/* First we introduce the premises (2 main premises plus    */
/*  4 additional assumptions).                               */

{1}      1. forall b (Author(Tolstoy,b) or Author(Joyce,b)
              ==> exists v (Copy(v,b) & Owns(Sam,v))              Premise

{2}      2. forall v (Owns(Barbara,v) ==>
                exists b (Copy(v,b) & Author(Tolstoy,b))          Premise


{3}    3. forall v,b1,b2
           Copy(v,b1) and Copy(v,b2)  ==> b1 = b2                 Premise


{4}    4. (exists b)(Author(Joyce,b))                Premise

{5}    5. (forall b,p)
         (Author(Joyce,b) & ~(p=Joyce) ==> ~(Author(p,b)    Premise

{6}    6.  ~(Tolstoy = Joyce)                               Premise


/* Now the proof begins.  Prove first intermediate step  */

{1}    7. Author(Tolstoy,B) or Author(Joyce,B)
          ==> exists v (Copy(v,B) & Owns(Sam,v))            U.S., 1

{8}    8. Author(Joyce,B)                                   Premise

{1,8}  9. exists v (Copy(v,B) & Owns(Sam,v))                T 7.8

{1}   10. Author(Joyce,B) ==>
          exists v (Copy(v,B) & Owns(Sam,v))               Cond. Discharge 8,10

{1}   11. forall b Author(Joyce,B) ==>
                    exists v (Copy(v,B) & Owns(Sam,v))      U.G., 10

/* Now we prove 2nd intermediate step, using extra assumption 6.  */
```

```
{12}  12. Owns(Barbara,V1) & Copy(V1,B1)                     Premise

{2}   13. Owns(Barbara, V1) ==>
            exists b Copy(V1,b) & Author(Tolstoy,b)           U.S., 2

{2,12}  14. exists b Copy(V1,b) & Author(Tolstoy,b)           T 12,13

{15}    15. Copy(V1,B1) & Author(Tolstoy,B1)                  E.S. Premise

{3}     16. Copy(V1,B1) & Copy(V1,B2) ==> B1=B2               U.S., 3

{3,12,15}  17. B1=B2                                          T 12,15,16

{3,12,15}  18. Author(Tolstoy,B1)                            substitution of equals

{2,3,12}   19. Author(Tolstoy,B1)                            E.S. Discharge, 14,15,18

{2,3}      20. Owns(Barbara,V1) & Copy(V1,B1)
               ==> Author(Tolstoy,B1)                       Cond. Discharge, 12,19

{2,3}      21. forall v,b                                   U.G., 20
               Owns(Barbara,v) and Copy(v,b)
                 ==> Author(Tolstoy,b)

/* Now we prove the third intermediate step */

{2,3}      22. ~ Author(Tolstoy,B1) ==>
                   ~(Copy(V1,B1) & Owns(Barbara,V1))         T 20

{5}        23. forall p Author(Joyce,B1) &
               ~(p=Joyce) ==> ~Author(p,B1)                  U.S., 5

{5}        24. Author(Joyce,B1) & ~(Tolstoy=Joyce)
               ==> ~Author(Tolstoy,B1)                       U.S., 23

{25}       25. Author(Joyce,B1)                              Premise

{5,6,25}  26. ~Author(Tolstoy,B1)                            T 5,24,25
```

```
{2,3,5,6,25}
           27. ~(Copy(V1,B1) & Owns(Barbara,V1))           T 22,26

{2,3,5,6,25}
           28. forall v1
               ~(Copy(v1,B1) & Owns(Barbara,v1))           U.G., 27

{2,3,5,6}  29. Author(Joyce,B1) ==>
                forall v1 ~(Copy(v1,B1) & Owns(Barbara,v1))
                                                          Cond. Discharge, 25,28

{1,2,3,5,6} 30. Author(Joyce,B1) ==>
                 exists v (Copy(v,B1) & Owns(Sam,v) &
                  forall v1 ~(Copy(v1,B1) & Owns(Barbara,v1))
                                                           T 11,29

{31}        31.  Author(Joyce,B1)                          E.S. Premise,4

{1,2,3,5,6,31}
            32. exists v (Copy(v,B1) & Owns(Sam,v)
                 & forall v1 ~(Copy(v1,B1) & Owns(Barbara,v1))
                                                           T 30,31

{1,2,3,5,6, 31} 33. exists b,v (copy(v,b) & Owns(Sam,v)
                 & forall v1 ~(Copy(v1,b) & Owns(Barbara,v1))
                                                           E.G. , 32

{1,2,3,4,5,6}
               34.  exists b,v (copy(v,b) & Owns(Sam,v)
                  & forall v1 ~(Copy(v1,b) & Owns(Barbara,v1))
                                                           E.S. Discharge 4,31,33
```