# The Naive Physics Perplex

Ernest Davis

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**Abstract**

The "Naive Physics Manifesto" of Pat Hayes (1978) proposes a large-scale project of developing a formal theory encompassing the entire knowledge of physics of naive reasoners, expressed in a declarative symbolic form. The theory is organized in clusters of closely interconnected concepts and axioms. More recent work in the representation of commonsense physical knowledge has followed a somewhat different methodology. The goal has been to develop a competence theory powerful enough to justify commonsense physical inferences, and the research is organized in *microworlds*, each microworld covering a small range of physical phenomena. In this paper we compare the advantages and disadvantages of the two approaches.

Common sense is a wild thing, savage, and beyond rules.
— G. K. Chesterton, *Charles Dickens: A Critical Study*

## Three Scenarios

Consider the following scenario:

> **Scenario 1:**
> A gardener who has a valuable plant with a long delicate stem protects it against the wind by *staking* it; that is, by plunging a stake into the ground near the plant and attaching it to the stake with string. (Figure 1.)

We might not all manage to think up this contrivance, faced with this problem, but we can all understand how it works. This understanding is manifested in a number of different abilities:

We can give an *explanation* of the problem and the solution. That is, we can generate a text along the following lines: "The wind may bend the plant; the fragile stem, bent too far, may snap, killing the plant. But if the plant is staked, then the string holds it in place, preventing any extreme bending. The string, in turn, is held in place by the stake, which, being comparatively stiff, is not bent either by the wind or by the force of the wind against the plant as transmitted through the string, and, being stuck in the ground, remains upright."

We can *carry out* the plan, which involves both hand-eye coordination and also the reasoning ability to fill in implicit steps of the plan. For example, the string must be looped around the stake and the plant and tied. Since the plan, as given above, does not specify this step, the reasoner must infer it.

We can *adapt* this solution to other problems, or adapt it to give alternative solutions to this same problem. For example, plants are sometimes staked to prevent their breaking under their own weight. An alternative to staking may be to encircle the plant with a metal frame.

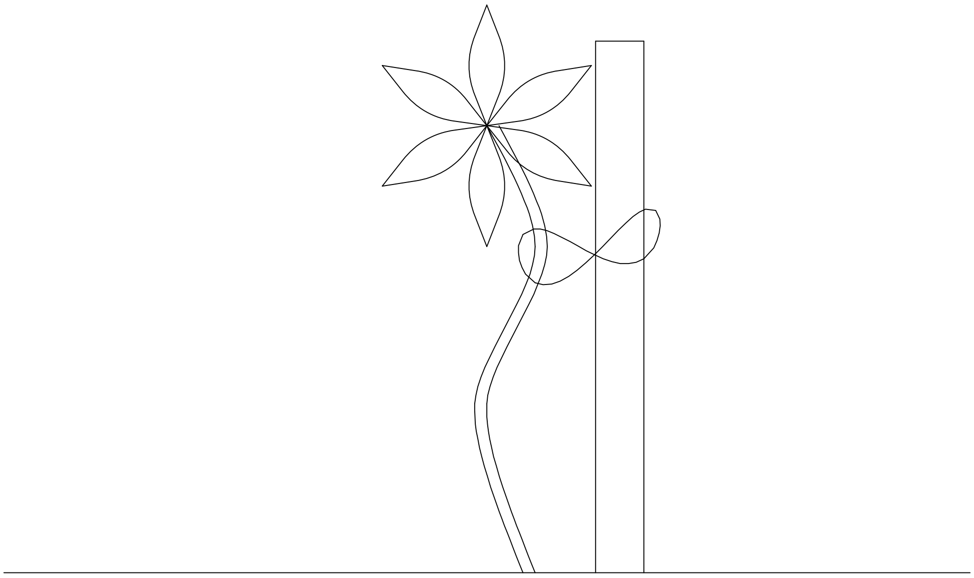


Figure 1: Staking a Plant

We can answer questions about *variants* of the plan. What would happen: If the stake is only placed upright on the ground, not stuck into the ground? If the string were attached only to the plant, not to the stake? To the stake, but not to the plant? If the plant is growing out of rock? Or in water? If, instead of string, you use a rubber band? Or a wire twist-tie? Or a light chain? Or a metal ring? Or a cobweb? If, instead of tying the ends of the string, you twist them together? Or glue them? Or place them side by side? If you use a large rock rather than a stake? If the stake is very much shorter than the plant? If the string is very much longer, or very much shorter, than the distance from the stake to the plant? If the distance from the stake to the plant is large as compared to the height of the plant? If the stake is also made out of string? Trees are sometimes blown over in heavy storms; can they be staked against this?

It would seem that the depth and power of our understanding is most readily exhibited by this last-mentioned ability of exploring variants. Over a limited class of plans, explanations and execution sequences can be canned, or generated by very narrow special-purpose techniques. Moreover, the difficulties in writing an adaptable text generator or plan executor are mostly those of natural language and of robotics, respectively; in practice, these issues swamp the problems of representation and reasoning. Adaptation and alternative application of plans certainly shows understanding, but may require a level of ingenuity that is not always reasonable to expect. But anyone with an understanding of the scenario should certainly be able to say something about how things change or stay the same under small changes of the situation or the plan; and, conversely, so many different variations can be hypothesized that intelligent answers can only be attained with some large degree of understanding.

Let us broaden our view by considering two more scenarios, with variants.

**Scenario 2:** (due to Leora Morgenstern (private communication))
In baking cookies, once you have the cookie dough prepared, you first lightly spread flour over a large flat surface; then roll out the dough on the surface with a rolling pin; then cut out cookie shapes with a cookie cutter; then put the separated cookies separately

onto a cookie sheet and bake.

What happens if: You do not flour the surface? You use too much flour? You do not roll out the dough, but cut the cookies from the original mass? You roll out the dough but don't cut it? You cut the dough but don't separate the pieces?

What happens if the surface is covered with sand? Or covered with sandpaper? If the rolling pin has bumps? or cavities? or is square? If the cookie cutter does not fit within the dough? What happens if you use the rolling pin just in the middle of the dough and leave the edges alone? If, rather than roll, you pick up the rolling pin and press it down into the dough in various spots? Ordinarily the cutting part of the cookie cutter is a thin vertical wall above a simple closed curve in the plane; suppose it is not thin? or not vertical? or not closed? or a multiple curve? If the cuts with the cutter overlap?

Does the dough end up thinner or thicker if you exert more force on the rolling pin? If you roll it out more times? If you roll the pin faster or slower? Do you get more or fewer cookies if the dough is rolled thinner? If a larger cookie cutter is used? If there is more dough? If the cuts with the cutter are spread further apart?

**Scenario 3:**
The following experiment is described in (Shakhashiri 1985) for estimating absolute zero using household objects. Prepare a pot of boiling water and a pot of ice water. Take an empty graduated baby bottle, complete with nipple attached, and submerge it (using tongs) in the boiling water. After a few minutes, when it has stopped bubbling, remove it and plunge it rapidly under the ice water. Water will then stream into the baby bottle through the nipple, as the gas contracts. (Actually, the nipple collapses; to allow the flow of water, you have to manipulate the nipple.) When the flow of water stops, the volume of the water that has entered the bottle may be measured by holding the bottle right-side up; the final volume of the gas at 0° C may be measured by holding the bottle upside down. The initial volume of the gas at 100° C is the sum of the final volume of the gas plus the volume of the water. By doing a linear extrapolation between these two values to the point where the volume of the gas would be zero, one can find the value of absolute zero.[1] (Figure 2).

What would happen: If the bottle is immersed only very briefly in the hot water? Or only very briefly in the cold water? If it is laid on top of the pots of water rather than immersed in them? If the bottle is left in the outside air for a long time between being in the hot water and being in the ice water? If the bottle has an open end with no nipple? If the nipple has no hole? If the bottle has other holes besides this nipple? If the bottle is opaque? If you use containers with air at 100° and 0° rather than water? If the quantity of ice water in the second pot is very small? very large? or if the quantity of hot water in the first pot is very small or very large? If the bottle is coated with styrofoam? If the bottle is opaque? If the bottle is not graduated? Why is the following not a reasonable experiment: "Take a volume of gas in your hands; cool it; and see how much it shrinks."

Additional problems of this flavor in commonsense reasoning can be found in (Miller and Morgenstern 1998).

[1] I tried this experiment three times. Twice, the entire baby bottle collapsed under the pressure in the cold water. The one time it ran successfully, it gave a value of −300° C for absolute zero, the true value being −273° C — not bad, for a baby-bottle experiment.

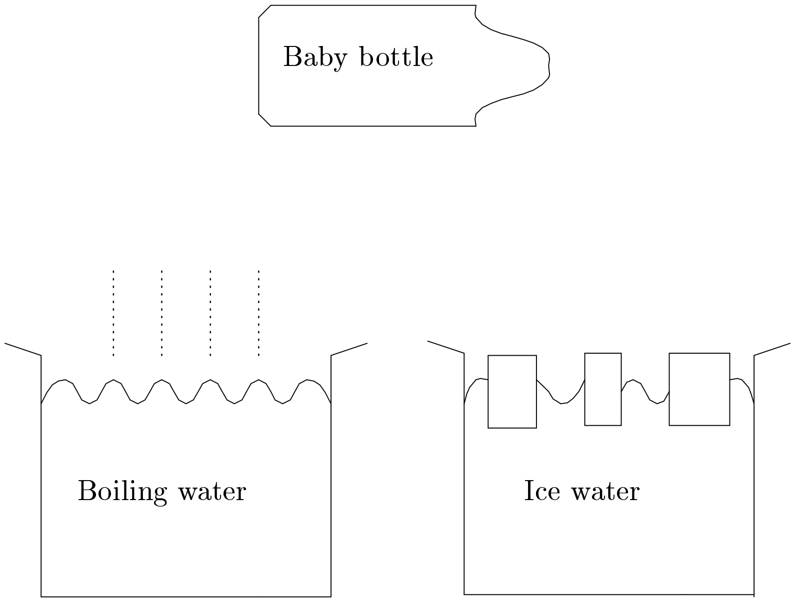



Figure 2: Determining Absolute Zero

## Commonsense Physics

These three scenarios above exhibit a number of characteristic properties:

They rely almost entirely on *commonsense knowledge*; that is, knowledge acquired informally at an early age, rather than explicitly taught. Scenario 3 requires an understanding of the thermal expansion of gasses, which is usually "book learning". All other aspects of this scenario, and all aspects of scenarios 1 and 2 are commonsensical. A naive subject who has been introduced to thermal expansion should be able to answer almost all the variant questions.

Quantitative relations are important; recall such questions as, "What happens if the string is much shorter than the distance from the stake to the plant?" or "What happens if the quantity of cold water is very small?" However, precise quantitative values are rare and textbook style equations are practically non-existent, with the exceptions, again, of the values 0°C and 100°C and the linear equation of thermal expansion.

Similarly, geometric properties and relations are important: The string must encircle the stake and the plant. The bottle must not have holes other than the nipple and must be immersed in the water. But no precise geometric descriptions are given or needed.

Each scenario involves a range of types of materials and processes. Scenario 1 involves the somewhat flexible plant, the gaseous wind, the rigid stake, the very flexible string, and the penetrable earth. Scenario 2 involves the malleable cookie dough and the rigid rolling pin, cookie cutter, and surface. Scenario 3 involves the solid baby bottle, the liquid water, and the gaseous air.

All three scenarios involve the manipulatory powers of an agent. Scenario 3, though not scenarios 1 and 2, also involves perceptual powers. The facts that the experimenter cannot simply cool a volume of gas that he holds in his hands, or that he cannot easily measure quantities in an opaque or ungraduated bottle, must be understood for these alternative experimental designs to be rejected.

All three scenarios lie outside the range of current automated reasoners. Since I have in the past (Tuttle 1993) been accused of giving an overly rosy impression of the state of the theory of automated commonsense reasoning, let me stress this point: *As far as I know, no one currently knows how to automate these inferences nor how to represent the knowledge used in them. I do not believe that this will be known any time in the near future.* The purpose of these three example scenarios is to indicate a direction for study and an ultimate goal, not to illustrate the capacities of existing programs or theories.

## The Naive Physics Manifesto

Commonsense physical reasoning was first and most famously promoted as a domain for AI research by Pat Hayes (1978) in the "Naive Physics Manifesto".[2] That paper advocated a research programme of developing a formalization of naive physics satisfying the following four criteria:

**Thoroughness.** "It should cover the whole range of everyday physical phenomena."

**Fidelity.** "It should be reasonably detailed."

**Density**. "The ratio of facts to concepts should be fairly high." A dense formalization is necessary "to capture the *richness* of conceptual linking." "Formalizations that are not dense in this way ... are unsatisfactory since they do not pin down exactly enough the meanings of the tokens they contain."

**Uniformity.** "There should be a common formal framework for the whole formalization." Hayes expresses a preference for first-order logic or some extension thereof, but does not insist on it. What is critical, in his view, is that the representation have a clear interpretation.

All considerations of implementation, application, or inference strategy are to be deferred until the formalization is largely complete. "It is not proposed to make a computer program which can 'use' the formalism in some sense. For example, a problem-solving program or a natural language comprehension system with the representation as target. [Such programs] have several ... dangerous effects. It is perilously easy to conclude that because one have a program that *works* (in some sense), its representation of its knowledge must be more or less *correct* (in some sense). Regrettably, the little compromises and simplifications needed in order to get the program to work in a reasonable space or in a reasonable time can often make the representation even less satisfactory that it might have been." Hayes further remarks "The decision to postpone details of implementation ocan be taken as an implicit claim that the representation content of a large formalisation can be separated fairly cleanly from the implementation decision; this is by no means absolutely obvious, although I believe it to be substantially true." This last point, of course, is a central point of attack by such critics as McDermott (1987).

The large theory of naive physics is structured in terms of *clusters*, a cluster being a nexus of concepts tightly related by a rich collection of axioms. Hayes gives the following examples of clusters: "measuring scales", "shape, orientation, and direction", "inside and outside", "histories", "energy and effort", "assemblies", "support", "substances and physical states," "forces and movements", and "liquids". A large part of the paper is devoted to preliminary analysis of these various clusters. The companion paper "Ontology for Liquids" (Hayes 1985b) is an in-depth analysis of the "liquids" cluster.

The question of finding the proper organization into clusters is considered one of the key issues

[2]All quotations in this section are taken from (Hayes 1978). The published version of this is always cited as (Hayes 1979); however, I have never actually set eyes on this, and I don't know what changes may have been made before publication. The later version (Hayes 1985a) is a substantially different paper.

in the enterprise:

> Identifying these clusters [of tightly associated concepts] is both one of the most important and one of the most difficult methodological tasks in developing a naive physics ... The symptom of having got it wrong is that it seems hard to say anything very useful about the concepts one has proposed ... But this can also be because of having chosen one's concepts badly, lack of imagination, or any of several other reasons. It is easier, fortunately, to recognize when one is in a cluster: assertions suggest themselves faster than one can write them down.

(I must confess that I personally have never attained the state of grace described in the last sentence above. In my experience, formalization is always a slow and delicate process, and a great deal of care is needed to avoid inconsistencies, unintended consequences, and gaps.)

Hayes proposes that the research programme be carried out by a committee. Each member of the committee will be assigned a particular cluster to formalize. The committee will meet from time to time in order to integrate their various efforts into a larger theory. This integration will no doubt require that formalizations of clusters be reworked, that new clusters be investigated, and that old ideas for clusters that prove to be useless be discarded.

One issue that Hayes discusses very little, rather curiously, is the choice of naive physics as a domain for study. He does say that "One of the good reasons for choosing naive *physics* to tackle first is that there seems to be a greater measure of interpersonal agreement here than in many fields," but he does not indicate what the other reasons might be. To my mind, the chief other advantages of naive physics as contrasted with, say, folk psychology or naive social science are:

- The power of "real" physics, the paradigm of a theory that is comprehensive, exact, and correct. The metatheoretic, mathematical, and logical structures have been extensively studied. Vast amounts of software carrying one or another type of computation in this domain have been implemented. Of course, naive physics is quite different from real physics; still, this give us an immense body of reliable knowledge on which to draw.
- Problems of intensionality and self-reference do not arise. Physics is a purely extensional theory.
- A broad range of practical applications.

Hayes' proposal derived in many key aspects from earlier proposals of John McCarthy's (1968). In particular, the choice of commonsense knowledge as subject matter, the idea of developing knowledge representations independently of implementation, and the choice of first-order logic as a representation language are all taken from McCarthy's previous work. What is chiefly new in Hayes' manifesto is the proposal to restrict the focus to naive physics, as opposed to other commonsense domains.

## Two Common Misconceptions

There are two common misimpressions of Hayes' proposal. The first is an understandable confusion. Seeing that the Naive Physics Manifesto and the Ontology for Liquids are full of formulas written in first-order logic and formal proofs, many readers have gotten the false idea that Hayes is proposing that a reasoning program should explicitly manipulate logical formulas using some general purpose theorem-proving method. Now, various people (e.g. (Moore 1982), (Kowalski 1979)) do indeed advocate this view, but Hayes does not, at least not in these papers.[3] He is, in fact, entirely agnostic as to how the knowledge should be implemented as data structures or what procedures should manipulate it. Hayes' proposal is to analyze naive physical reasoning at the *knowledge level* (Newell 1980), in terms that are independent of the particular computing architecture, algorithms, and data structure. First-order logic is chosen as a language to describe the knowledge level, precisely because it is a neutral one, that does not presuppose any particular form of implementation.

The intended relation between a logical domain theory and a reasoning program is similar to the relation between a programming language semantics and a compiler. The semantics specifies what the compiler should do; a compiler is correct if the semantics of the output code is compatible with the semantics of the source code. But one does not necessarily expect a compiler to be written in the abstruse formalisms of programming language semanticists. Similarly, the desired relation between a logical domain theory and a reasoning program is that the theory should *characterize* or *justify* the actions of the program, in the sense that some significant part of the results computed by the program corresponds to, or approximates, valid conclusions in the theory. But the internals of the program need not contain anything that looks like the theory.

For example, STRIPS-style planners can be characterized in terms of the situation calculus, in the following sense: Given a collection of actions in the STRIPS representation, you can construct a situation calculus theory defining the domain such that any plan output by the planner can be proven correct in the theory (Lifschitz 1986). Another example: A simulator that calculates solutions to gravitational motion by numerically solving the differential equations can be characterized in terms of a formal theory containing Euclidean space, real-valued time, and Newton's law of gravitation, in the sense that the output of the program *approximates* the conclusions of the theory. (Defining this sense of "approximates" exactly is a substantial undertaking, of course.)

One major difference between compilers, STRIPS, and gravitational calculation, on the one hand, and a general commonsense reasoner, on the other, is that the former programs are doing inference in a single direction with complete information or a narrow range of partial information, whereas a general reasoner, as we have discussed, should do reasoning in many different directions using whatever partial information it has. Therefore, it is therefore more critical in a commonsense reasoner to use a widely expressive and declarative representation and a flexible inference mechanism; hence, the interest in logical representations and symbolic deduction for implementing reasoning systems. But these considerations are largely irrelevant to Hayes' argument. Note that the success of formal programming-language semantics shows that logical analysis can be valuable even when the task being studied is narrowly focussed.

The second common misconception is a little more peculiar. There is a widespread misimpression that if geometric information is represented in first-order logic, then the primitives used must correspond to basic spatial terms in natural language. For example, people[4] will assert that the only logical representations of the situation in figure 3 are something like

left-of(a,b). left-of(b,c). left-of(c,d).
red(a). white(b). red(c). blue(d).

People sometimes go so far as to conclude from this supposition that retrieving the fact that the leftmost object is left of the rightmost, or retrieving the fact that block E is not in this line, will take time at least linear in the number of objects.

There is, of course, not the slightest truth in this. The following are all valid logical sentences,

[3] Even in the paper "In Defense of Logic" (Hayes 1977), the argument is just that a representation should have a well-defined semantics, and that many of the "alternatives" to logic-based representations being touted at the time did not.

[4] I have recently heard precisely this statement made in a public talk by a distinguished researcher in spatial reasoning.


Figure 3: Blocks to be represented

given a suitable semantics: (Take the origin to be the lower-left hand corner of block A and the unit to be the side of that block, with axes aligned as usual.)

place(c) = rectangle(point(3,0), point(5,0), point(5,1), point(3,1)).
red-pixel(pixel(4,0)).
empty(rectangle(point(1,0), point(2,0), point(2,1), point(1,1)).
$\forall_X$ block($X$) $\Rightarrow \exists_Y$ red($Y$) $\wedge$ distance($X, Y$) $< 2$.

In fact, with the exception of probabilistic distributions and "fuzzy" distributions over space, every representation of spatial or geometric information that I have ever seen can be straightforwardly expressed in a first-order logic over a universe of simple geometric entities. Indeed, in the great majority of representations in use, the ontology can be taken to be Euclidean space, and the language can be restricted to a constraint logic.[5] In particular, all of the representations of spatial information that are considered "diagrammatic" (Glasgow, Narayanan, and Chandrasekaran 1995) can be straightforwardly expressed in first-order logic over Euclidean geometry; and and all of the inferences considered in (Fleck 1996) can be justified in a Euclidean geometry, given a suitable statement of the physical axioms. I'm not claiming, of course, that there is necessarily anything to be gained from translating non-logical representations into logical representations, merely that these alternative representations do not express any kind of information that can't be expressed in first-order logic. There are types of non-spatial information that are impossible or extremely awkward to express in first-order logic, such as uncertain knowledge, meta-knowledge, and propositional attitudes. But very, very few declarative representations of spatial information involve any of these problem. (Non-declarative representations, such as procedural representations, or representations in terms of the state of a neural network, do not, of course, translate well into first-order logic.)

# Difficulties with the Manifesto

Hayes' Manifesto was much admired and widely discussed, but it was hardly followed. The committee never met, the theories were never codified. There has, of course, been a great deal of work in "qualitative physics" but this has a quite different flavor from Hayes' proposal; it is algorithmic rather than declarative, and is increasingly concerned with specialized applications rather than commonsense reasoning (Weld and de Kleer 1989), (Iwasaki 1997). Even interpreting the manifesto fairly broadly, it would be difficult to think of more than a dozen AI researchers who have done the kind of work in physical reasoning that Hayes has in mind, while interpreting it narrowly, one could certainly argue that the manifesto and the Ontology for Liquids are the only two papers ever written that fit into Hayes' programme.[6]

No doubt the main reason for this neglect is simply that life is short, the project is large, and researchers have had other things to do that seemed more pressing. But, besides this, the project

[5] A constraint logic is the conjunction of atomic ground sentences, without negation, disjunction, or quantification.
[6] (Schmolze, 1986) should also be mentioned.

as Hayes outlines it has fundamental difficulties in its conception, and researchers who try to follow in Hayes' footsteps soon find themselves head to head with these obstacles.

It is not really clear what, precisely, Hayes means by "naive physics". The Naive Physics Manifesto is for the most part written as if "naive physics" were a clearly defined body of knowledge — comprehensive in scope, universal across people, consistent, and essentially uninfluenced by science. More than once, Hayes claims that some specific concept or distinction is or is not a part of "naive physics", apparently in an absolute sense:

> Naive physics is pre-Galilean. I can still vividly remember the intellectual shock of being taught Newtonian "laws of motion" at the age of 11. It is interesting to read Galileo"s "Dialogue Concerning the Principal Systems of the World" (1632) where he argues very convincingly, from everyday experiences, that Newton's first law must hold. But it takes a great deal of careful argument ... (Hayes 1978)
>
> I have deliberately not distinguished between mass and volume. I believe the distinction to be fairly sophisticated. (Hayes 1985a)
>
> In making predictions, there is a distinction which seems crucial between events that "just happen" (such as fallings) and events which require some effort or expenditure of energy (such as rocks flying through the air). ... Such a distinction runs counter to the law of conservation of energy, and I think quite correctly so for naive physics (or we could say merely that the intuitive notion of "effort" does not exactly correspond to the physical notion of "work".) (Hayes 1978)

Now, Hayes does not, of course, actually believe in such an absolute, monolithic theory. He specifically acknowledges and discusses individual differences in the system of naive physics beliefs. Further, the first quote above at least implicitly acknowledges that an individual's beliefs may be inconsistent. (If Newton's first law can be derived by Socratic argument and Gedanken experiments from memories of everyday experience, but is also explicitly denied in naive physics, then the closure of the individual's beliefs under "reasonable argument" is inconsistent.)

Trying to define an absolute "Naive Physics" raises many difficulties. First, naive physics is supposed to be what naive subjects believe about the physical world. But, as is well known, the concept of "belief" is ambiguous and slippery, with many different possible interpretations. "$A$ believes $\phi$" may mean that $A$ will spontaneously assert $\phi$; that $A$ will immediately assent to $\phi$; that $A$ will assent to $\phi$ after Socratic interrogation; that $A$ will assent to statements that logically entail $\phi$; that the best explanations of $A$'s actions at the knowledge level involve the assumption that $A$ is using $\phi$ in the course of reasoning; or that $A$'s actions are more sensible given that $\phi$ is true than given that it is false. Which is intended here?

Second, the problem of defining "belief" is made more difficult by the constraint that we are interested only in "naive" beliefs, not in beliefs that are formally taught, but that the most readily available subjects — the researchers themselves — tend to be people with substantial training in formal science and mathematics. It is not clear how we can tease out a true "naive physics" from later accretions of formal physics.

Third, physical reasoning depends critically on spatial knowledge and spatial reasoning that is difficult or impossible to express in ordinary language. For instance, we all know how a screw is shaped, and we all have some understanding of the relation between the shape of a screw and its functions. (This understanding is most easily demonstrated through the methods of considering variants. For instance, it is easy to see that a small pit in the surface of the screw will probably have little effect on its behavior, whereas a small bump is likely to be much more troublesome.) However, it is not easy to *describe* verbally the shape of a screw or to *explain* verbally the connection between its shape and its behavior, without using a technical vocabulary unintelligible to most naive subjects.

This is probably the chief disadvantage of physics as opposed to other commonsense domains as a testbed for studying commonsense reasoning.

Fourth, "naive physics" probably varies substantially between people (though Hayes may well be right that it differs less than other branches of commonsense knowledge). Due to the vagueness in defining "naive belief", it is difficult to be very precise about this. But one can certainly see it in cross-cultural comparisons. For instance, many people in various times and places have attributed intentions and mental states to inanimate objects. In modern Western culture, this is not part of even a "naive" system of beliefs.

One can try to work around this difficulty by observing that people's beliefs are at least close enough to enable them to communicate, and defining the "naive physics" we are looking for as the beliefs that are common knowledge within the community. For instance, a subject who believes that one sees an object using reflected light and another subject who believes that one sees an object using emanations from the eyes will nonetheless agree that one cannot see through an opaque object. Therefore, if the community contains large numbers of believers in both theories, the naive physics would contain the belief that one cannot see through an opaque object, but would exclude both the theory of reflected light and the theory of ocular emanations as speculative theory. Is it possible to develop a naive physics rich enough to support commonsense inferences on the basis of this kind of common knowledge? The question is important, but very difficult. Certainly, the central role played by inarticulable spatial knowledge makes this problem more difficult.

Finally, it is not clear that an individual's beliefs are consistent. It depends in part, of course, on how "belief" is defined. An inconsistent belief set cannot be expressed in a single theory in any standard logic (or indeed in most non-monotonic logics).

The result of this unclarity is that the researcher really has no way of determining whether a given concept, distinction, or rule is to be considered a legitimate element of "naive physics." Does the concept "surface area" exist in naive physics? Or the concept of an object being "awkward to handle"? Or the distinction between heat and temperature? How is one to judge? Pat Hayes (personal communication) tells a story of engaging in a two hour debate over whether a picture hanging on the wall of a room can be said to be "in" the room. Such minutiae are essentially unavoidable in this approach to formalization.

A particularly difficult issue to judge is the appropriate level of generality. Consider the rule in the cookie-baking domain, "The thinner you roll the dough, the more cookies you get." Now, this fact can be expressed directly in this form. Alternatively it can be derived from the considerations that

1. The volume of the cookie dough is fixed. In particular, it is not affected by rolling it out.
2. The volume of a region is equal to its area times its average thickness.
3. The number of regions of fixed shape $A$ that can be placed disjointly within a region $R$ tends to increase with the area of $R$. (Note that this is a plausible inference, rather than a sound rule.)
4. In cutting cookies out of rolled-out dough, each cookie is a cross-section of the dough on a vertical axis, and no two cookies overlap.

Or one can use rules at an intermediate level of generality (e.g. replace (2) by the more specific rule, "For a fixed quantity of malleable stuff, the thinner it is spread on a surface, the larger the area it covers"), or at a higher level of generality (e.g. derive (3) from a definition of volume as an integral.) Using the more general formulation usually has the advantages of covering more physical situations, and clarifying the relations between them, but each level of generality seems less and less "naive". How do we choose among them?

Some will argue that terms like "volume", "average" and "cross-section", which are used in our second set of rules above, are formally learned in school and therefore are not part of a naive theory. Now, certainly the more specific rule, "The thinner you roll the dough, the more cookies you get," may be one that a child learns first, before any more general formulation, and it may be a rule of thumb that someone baking cookies regularly calls upon, without doing deeper thought. But it seems to me that an intelligent person will soon see the connection between this fact and the facts that, if you want to cover a table top with books, you will do better to lay them flat and not to stack them; that a can of paint will cover a small area more thickly than a large area; and, at a further remove, that the more people are sharing a pie, the smaller each person's piece. To express the general rules that underlie these particular instances, you will almost certainly have to call on concepts that are so close to the standard ones of "volume", "average", and so on, that the distinction is hardly worth making. (Quite likely, the naive reasoner is reasoning by analogy or using case-based reasoning, rather than using an explicit generalization, but in that case these same concepts will be needed to find the dimensions of similarity between the cases. Thus the necessary expressivity of the object language is largely independent of the mode of reasoning.) Therefore, despite the association of these terms with the classroom and textbook, it seems difficult to me to justify automatically excluding these concepts from a naive understanding. I should say, rather, that teaching these in the classroom is, or should be, mostly a matter of putting concepts that are already understood at the commonsense level into a rigorous setting.

## Microworlds: A modified methodology

One way out of these difficulties begins by arguing as follows: Whatever the actual content of people's individual theories, they will almost all come up with the same or similar answers over a large collection of commonsense problems. A program will achieve common sense if it gives the same answers to the same problems. Therefore, *any* theory that allows commonsense problems to be stated and solved will do. In other words, we are looking for a competence theory for solving commonsense problems. Note that we have substantially shifted our ultimate goal. Before, we were talking about expressing a body of knowledge; now we are talking about justifying a collection of inferences.

The second change that we will make is to focus on *defining a model*[7] rather than stating an axiomatic theory. The argument for this change is as follows: As discussed above, our main goal in formalizing theories is to characterize or justify the actions of reasoning programs, rather than to be implemented directly as a rule base. But the relation of "justifying" a particular inference or "characterizing" a particular program is a property of a model, not of a specific axiomatization of that model. If a model can be axiomatized in two equivalent ways, the two axiomatizations support the same inferences. Therefore, our primary concern will be defining a model, and thus determining the class of true statements and valid inferences in the model. Secondarily, we are interested in defining a formal language, which delimits expressive range, the class of facts that can be expressed. In this approach, axiomatizations are only of subsidiary interest; they help clarify the model and they are useful in verifying that a given inference is indeed supported by the model.

A third change is in the way in which the project is divided into parts. Hayes' goal is to express a theory, so a natural subset of the project is a coherent subset of the theory; that is, a cluster of concepts and axioms. The new goal is to characterize inferences, so a natural subset of the project is a *microworld*: an abstraction of a small part of physical interactions, sufficient to support some

[7] I will generally use "model" in this paper in the sense common in physical reasoning research: A model is an abstract structure that mirrors some of the significant properties of a physical microworld. This is somewhat different from the meaning of the term in metalogic. When I need the term from metalogic, I will say so specifically.

interesting collection of inferences.[8]

A few examples of microworlds:

1. The roller coaster world (de Kleer 1977). The world consists of a point object and a one-dimensional track in a vertical plane. The state of the world is either the position and velocity of the object along the track, or the distinguished state "FELL-OFF". The motion of the object is governed by Newton's law, with gravity and inertia. The microworlds of (Forbus 1980) and (Sandewall 1989) are similar.
2. Component-based electronics (de Kleer and Brown 1985). The world consists of resistors, capacitors, inductors, power sources, etc. connected in a circuit. The state of the world at any moment is the voltage at every node and the current through every arc. The world changes dynamically following component characteristics.
3. Rigid object kinematics. The world consisting of solid, rigid objects, constrained by the rules that the shape of an object is fixed, that it moves continuously and that two objects do not overlap.
4. Rigid object dynamics. The world consisting of medium-sized solid, rigid objects, moving in a uniform graviational field, and interacting through normal forces, friction, and impacts, above a fixed ground.
5. Kinematics of solid objects and a liquid. The world consisting of solid objects and some quantity of a liquid. The solid objects are constrained by the rules that their shape is fixed, they do not overlap, and their motion is continuous. The liquid is constrained by the rule that its volume is constant, it moves continuously, and it does not overlap the solid objects.

Note the difference from clusters. Of Hayes' clusters, only "liquids" is close to being a microworld, and even this would almost certainly have to be changed to "liquids and solids" (under some specified set of physical laws), as there are very few commonsense inferences that involve only liquids with no solid boundaries.

We may also contrast microworlds with reasoning architectures, such as QP (Forbus 1985) or ENVISION (de Kleer and Brown 1985). QP and ENVISION do not incorporate any particular physical theory. Rather, each such architecture provides a collection of basic ontological sorts, a restricted language in which physical theories of certain types can be stated, and an algorithm for carrying out certain types of inference. For instance, the basic sorts in QP include time instants, time intervals, parameters, and processes. The QP language supplies primitives symbols for "direct influence" and "indirect influence", which have a fixed interpretation. The algorithm carries out qualitative envisionment.

Thus, the development of this kind of program is orthogonal to the microworlds methodology. The microworlds approach focusses on developing specific physical theories; programs such as QP and ENVISION focus on developing techniques that apply across a range of physical theories.

Another change from Hayes' project is in the attitude toward beliefs that are commonsensical but false. These can be divided into three categories:

1. Beliefs that are approximately correct in everyday contexts. For example, the belief that a moving object will come to a halt if no force is applied. This rule, which contradicts Newton's first law, holds for most objects in most terrestrial circumstances.

[8] The term "microworlds" goes back in AI research at least as far as the early 70's (Minsky and Papert 1970). These microworlds, however, had a quite different purpose; they were simplified testbeds for exploring such issues as inference, search, planning, learning and so on. CYC, in its later versions (Lenat and Guha 1993), is the most notable recent exemplar of the use of microtheories. These are axiom-based, rather than model-based.

2. Logical consequences of rules in (1). For example, the belief that if a torque is applied to a gyroscope, the gyroscope will rotate along the axis of the torque. This is just a special case of the general rule, "If a torque is applied to an object, then the object rotates along the axis of the torque," which holds for most objects but not gyroscopes.

3. Beliefs that are just plain wrong, without either of the above justifications. For instance the belief that an object that has been moving along a circular track will continue to move in a circle once it is free of the track (McCloskey 1983).

A competence theory of commonsense reasoning system may well include beliefs of category (1); indeed, at some level it must, unless we plan to base it on relativistic quantum mechanics. This is justified as a trade-off of accuracy for speed and simplicity. We are therefore also likely to get beliefs of category (2), unless we can block them all using qualification conditions, which is unlikely. The question is whether there is any point in including beliefs of category (3). For Hayes' project, where the ultimate aim is a cognitive model of a naive reasoner, presumably they should be included. Likewise, if we were studying the process of *learning* physical theories, we would have to expect that sometimes the theories being considered are entirely off-base. In a competence theory of reasoning, however, since these add nothing to competence, they should be excluded. For this reason, in the new approach we speak of "commonsense physics" rather than "naive" physics.

Putting all this together, we arrive at a methodology along the following lines[9] (Figure 4)

1. Select a microworld: a well-defined, fairly small, range of physical behaviors.
2. Collect a corpus of inferences in the domain that are both physically correct and would be broadly agreed upon as commonsensically obvious.
3. Develop
   a. A formal model of the domain.
   b. A language of primitives with semantics defined in the model.
   c. An axiomatization of the model expressed in the language.
4. Demonstrate that many of the inferences in (2) can be expressed in the language (3.b) and justified in the model (3.a). A formal proof from the axiomatization (3.c) may be helpful here.
5. Develop algorithms or programs that can be justified in terms of this model, and show that some significant class of commonsense inferences can be carried out efficiently.
6. Work toward broadening theories and merging multiple theories together.[10]

Two recent projects of a similar flavor should be mentioned. Ken Forbus (1998) presents a characteristically ambitious proposal to construct a library of foundational qualitative domain theories, containing on the order of 10,000–100,000 axioms, encoded in first-order logic, I have not been able to get detailed accounts of these domains, and so have not been able to make comparisons with the issues discussed here. In a different direction, a recent triad of papers (Lifschitz 1997), (Morgenstern

[9]The methodology described here is my own personal view (Davis 1990); however, little if any of this is original to me. Particularly significant discussions of this kind of methodology in this direction, besides (Hayes 1978), include (McCarthy 1968), (McCarthy and Hayes 1969), (McDermott 1978), (Newell 1980), and (Charniak and McDermott 1985). (Halpern and Vardi 1991) similarly argues from a shift from an axiomatic to a model-based analysis in automated reasoning.

[10]John Tsotsos pointed out to me that this list should have an additional item of developing techniques to learn or acquire this knowledge. This is undoubtedly correct, but I find the idea of trying to learn this material automatically too terrifying to contemplate.

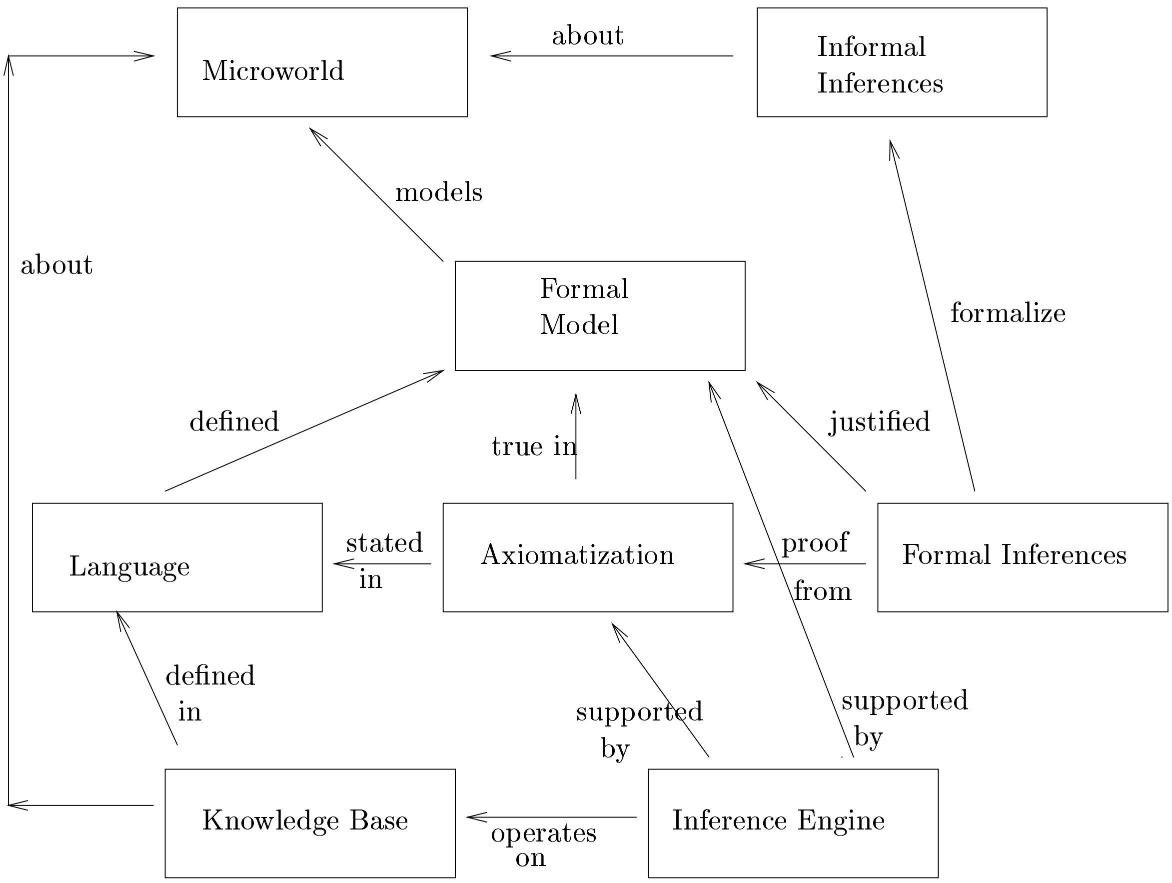



Figure 4: Methodology

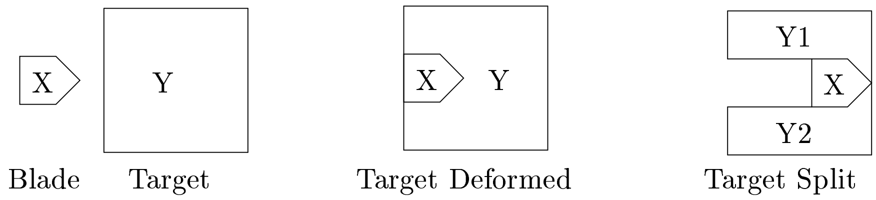



Figure 5: Mutable Object Theory

1997b), (Shanahan 1997) dealing with the "egg-cracking" problem attempts a new method for advancing the state of the art in commonsense reasoning. A complex but narrowly defined inference, like those used in our original three scenarios, is put forward; separate researchers work independenly on developing their own theories for justifying this particular inference; and then hopefully the insights gained from this example can be combined and applied to the next example.

## A sample microworld: the kinematics of cutting solid objects

At this point, it may be helpful to give a rather detailed description of one microworld, for illustration. The example I will use is a kinematic theory of cutting solid objects (Davis 1993). Relative to the state of the art in formalizing physical theories, this is a fairly complex and sophisticated example.

### Microworld

The microworld is the kinematics of cutting rigid solid objects. That is, the world consists of solid objects moving continuously through space on arbitrary paths. The shape of any object is constant except when the objects is being cut. Objects are not created or destroyed except at the moment when one object is sliced through.

The process of cutting is modelled as if the blade annihilates the material of the target as it penetrates. When the annihilation of material leaves the target disconnected, it falls into two or more new objects (Figure 5.) This model is rich enough to support many manners of cutting: slicing through, stabbing through, filing down, or carving a cavity.

The model does not support the intuitive distinction between "cutting a small piece off of object $A$," where the identity of $A$ survives in a smaller shape; and "slicing object $A$ into objects $B$ and $C$" where $A$ ceases to exist and $B$ and $C$ come into existence. All cases where an object is split are considered in the second category, no matter how small the piece being split off.

The model does not support any theory of *dynamics,* in the sense of forces, energy, and such. For that reason, it does not incorporate any shape constraints on the blade, such as that it be sharp or serrated, or on the motion, such as that it involve sawing back and forth, as these would be arbitrary and inadequate in the absence of a dynamic theory. Similarly, the model does not incorporate the deformation of material that generally takes place in actual cutting; material is

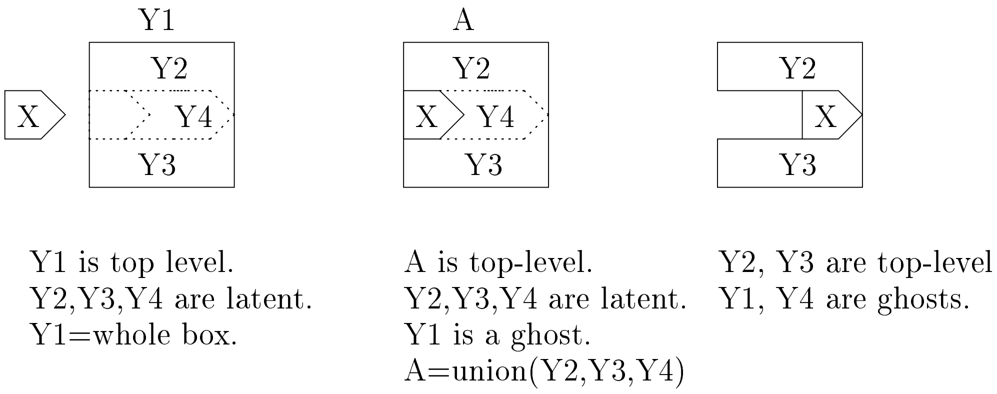



Figure 6: Chunk Theory

simply and irreversibly vaporized.

## Ontology

In developing an ontology, we begin by constructing a model of time and space. We model time as the real line. A *situation* is an single instant of time. A *fluent* (McCarthy and Hayes 1969) is an entity whose value changes through time. For example, "the President of the United States" is a fluent whose value in 1791 was George Washington, and in 1998 is Bill Clinton. We model space as three-dimensional Euclidean space. Other models of space and time might be possible, if they support the following concepts with suitable properties: earlier/later times, spatial regions, connectivity, rigid motions, continuous rigid motions, set difference of regions, and overlap of regions.

We can now formulate two alternative construals of the above model of cutting. The first, more straightforward, approach construes the world in terms of objects, as above. The shape of an object $O$ is a fluent that changes through time as $O$ is cut and material is removed. When the shape of $O$ become disconnected, $O$ ceases to be "present" and becomes a "ghost"; and two new objects $O1$ and $O2$ cease to be "ghosts" and become "present." Thus, each object can undergo three types of change during its lifetime: It is originally created by being sliced off some parent object; then its shape is gradually modified as it is cut away; then it is destroyed when its shape is split.

The second construal focusses on chunks of material. A "chunk" is a physically connected piece of material; it is the part of an object that fills some connected, topologically open region. At any given moment, an object has one chunk that is "top-level", meaning that its shape is exactly the shape of the object, and many chunks that are "latent", meaning that their shape is a proper subset of the shape of the object. The latent chunks are, so to speak, waiting for a suitable cutting process to carve them out and make them top-level for their moment in the sun. A chunk of a target is "destroyed" as soon as it is penetrated by the blade. Thus the process of cutting involves the continual destruction of an infinitude of chunks which now have some of their material annihilated. At most instants, a single new chunk becomes top-level for an instance; occasionally, at the instants when the object is split, two new chunks become separately top-level. The shape of a chunk is constant. Thus, in this theory there is only one kind of change: an active chunk (i.e. one that is either top-level or latent) becomes a ghost. (Figure 6.)

The advantage of the chunk approach is that there is now only one type of change: the annihilation of material, formalized as the destruction of chunks. Sometimes this annihilation leaves a single

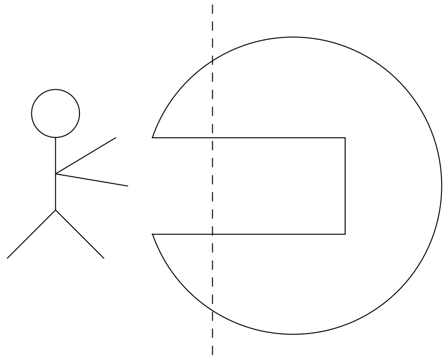


Figure 7: Carving one object or two?

top-level chunk, sometimes more than one, but the two essentially "look the same" from the point of view of the model. This can be useful in cases like that illustrated in figure 7. A sculptor is carving away at a pair of stone pieces, of which he can see only the nearer parts. In the object theory, this situation is difficult to describe, because he cannot know whether this is in fact one object or two; it depends on whether the two pieces are connected, which he cannot see. Worse, the two pieces may originally be a single object and then become two, when someone splits the connection behind the scene. However, assuming that the structure is fixed, it should make no difference to the sculptor whether the two pieces are connected, and in the chunk theory it doesn't. The chunks in the area visible to the sculptor are the same whether or not they are connected behind.

These two theories can be proven to be equivalent, under certain minor regularity conditions (Davis 1993). (These exclude scenarios in which an object is sliced infinitely many times in a finite time interval, and other such *outré* and non-physical possibilities.)

We can then define the *process* of cutting: Object $A$ is cutting object $B$ at time $T$ if, for every previous time $T'$, there was material in $B$ at $T'$ that $A$ overlaps in $T$. (This is a minor improvement on the definition in (Davis 1993).) Somewhat more arbitrarily, we can individuate a cutting *event*: A cutting event of $B$ by $A$ occurs over time interval $I$ if $A$ is cutting $B$ throughout $I$ but not throughout any proper superinterval of $I$.

## Language and Axiomatics

Tables 1 and 2 display languages sufficient to express the basic concepts of the two theories, and tables 3 and 4 show the basic physical axioms of the two theories. Basic geometric and temporal primitives, such as "image", "<", "continuous", and so on, are defined relative to Euclidean space and real-valued time, as indicated in table 2. The axioms are written in a sorted first-order logic. To shorten the notation, we use fluent functions as predicates with an additional situational argument. Thus, for instance, the statement "Object $O$ is material in situation $S$," can be expressed equivalently either in the form "holds($S$,material($O$))" or "material($O, S$)".

| Sort | Letter |
|---|---|
| Point | $X$ |
| Spatial regions (set of points) | $R$ |
| Rigid mappings | $M$ |
| Temporal situations | $S$ |
| Fluents | $F$ |
| Objects | $O$ |
| Chunks | $C$ |
| Either object or chunk | $Q$ |

Table 1: Logical Sorts

Temporal:

holds$(S, F)$ — Predicate. Boolean fluent $F$ holds in situation $S$.
value_in$(S, F)$ — Function. Value of fluent $F$ in situation $S$.
$S1 < S2$ — Predicate. Situation $S1$ precedes $S2$.
just_before$(S, F)$ — Predicate. Boolean fluent $F$ holds in an open interval ending in $S$.

Spatial:

$X \in R$ — Predicate. Point $X$ is in region $R$.
$R1 \subset R2$ — Predicate. Region $R1$ is a proper subset of $R2$.
$R1 - R2$ — Function. The interior of the set difference of $R1$ and $R2$.
intersect$(R1, R2)$ — Predicate. Region $R1$ intersects $R2$.
$\emptyset$ — Constant. The empty region.
good_shape$(R)$ — Predicate. Region $R$ is non-empty, open, bounded, connected, and equal to the interior of its closure.
image$(M, R)$ — Function. The image of region $R$ under mapping $M$.
continuous$(F, S)$ — Predicate. $F$ is a continuous function of time at situation $S$. $F$ is a fluent whose value in each situation is a rigid mapping.
connected_component$(R1, R2)$ — Region $R1$ is a connected component of $R2$.

Physical: Primitive Symbols

material$(Q)$ — Function. The fluent of object or chunk $Q$ being material.
placement$(Q)$ — Function. The fluent of the mapping from the shape of $Q$ to the place of $Q$.
shape$(O)$ — Function. The fluent of the point set occupied by $O$ in a standard orientation.
cshape$(C)$ — Function. The time-invariant shape of chunk $C$.

Physical: Defined Symbols

ghost$(Q)$ — Function. The fluent of $Q$ being a ghost.
place$(Q)$ — Function. The fluent of the region occupied by $Q$ in situation $S$.
blade_swath$(S1, S2, O)$ — Function. The swath cut by blades between situations $S1$ and $S2$, relative to the coordinate system attached to object $O$.
destroyed$(S, O)$ — Function. Object $O$ is destroyed at time $S$.
top_level$(C)$ — Function. The fluent of chunk $C$ being top-level.
sub_chunk$(C1, C2)$ — Predicate. Chunk $C1$ is (non-strictly) a sub-chunk of $C2$.

Table 2: Non-logical primitives

Definitions of Object Theory

OD.1 ghost$(O,S) \Leftrightarrow \neg$material$(O,S)$.
(Definition of ghost: An object is a ghost iff it is not material.)

OD.2 place$(O,S)$ = image(placement$(O,S)$, shape$(O,S)$).
(Definition of place: The region occupied by $O$ in $S$ is the image of its shape under its placement.)

OD.3 $X$ $\in$blade_swath$(S1,S2,O) \Leftrightarrow$
$\exists_{S3,OB}$ $S1 \leq S3 \leq S2 \wedge OB \neq O \wedge$ image(placement$(O,S3)$,$X$) $\in$ place$(OB,S3)$.
(Definition of blade-swath: The blade-swath between $S1$ and $S2$, relative to $O$, is the region swept out by all blades between $S1$ and $S2$, as measured from a coordinate system attached to $O$.)

OD.4 destroyed$(S,O) \Leftrightarrow$ [just_before($S$,material$(O)$) $\wedge \neg$good_shape(shape$(O,S)$)]
(An object is destroyed at $S$ if it existed up to $S$, but became disconnected or null at $S$.)

Axioms of Object Theory

OB.1 [material$(O1,S) \wedge$ material$(O2,S) \wedge O1 \neq O2] \Rightarrow$
$\neg$intersect(place$(O1,S)$, place$(O2,S)$).
(Two material objects do not overlap.)

OB.2 [$S1 < S2 < S3 \wedge$ material$(O,S1) \wedge$ material$(O,S3)$ ] $\Rightarrow$ material$(O,S2)$.
(Objects do not change from material to ghost to material.)

OB.3 material$(O,S) \Rightarrow$ good_shape(shape$(O,S)$).
(Material objects have good shapes.)

OB.4 $\forall_{S,O}$ shape$(O,S) \neq \emptyset \Rightarrow$ continuous(placement$(O)$,$S$).
(The placement of object $O$ is continuous in any situation $S$ where the shape of $O$ is non-null.)

OB.5 [$S1 < S2 \wedge$ material$(O,S1) \wedge$ just_before($S2$,material$(O)$)] $\Rightarrow$
shape$(O,S2)$= shape$(O,S1)$ $-$ blade_swath$(S1,S2,O)$
(The material removed from $O$ between $S1$ and $S2$ is the blade-swath between $S1$ and $S2$ relative to $O$, plus boundary points.)

OB.6 [destroyed$(S,O) \wedge$ connected_component($R$,shape$(O,S)$)] $\Rightarrow$
$\exists_{OR}$ shape$(OR,S)$=$R$ $\wedge$ placement$(OR,S)$ = placement$(O,S)$ $\wedge$
just_before($S$,ghost$(OR)$) $\wedge$ material$(OR,S)$.
(If $O$ becomes disconnected or null at $S$, then each of its connected components become material.)

OB.7 [material$(O,S1) \wedge$ ghost$(O,S2) \wedge S1 < S2] \Rightarrow \exists_{S3 \in (S1,S2]}$ destroyed$(S3,O)$
(An object turns from material to ghost only if it is destroyed in the sense of OD.4.)

OB.8 [ghost$(O,S1) \wedge$ material$(O,S2) \wedge S1 < S2] \Rightarrow$
$\exists_{S3,O3}$ destroyed$(S3,O3) \wedge S1 < S3 \leq S2 \wedge$
connected_component(place$(O,S3)$, place$(O3,S3)$).
(An object can come into existence between $S1$ and $S2$ only if it is a connected component of some object $O3$ that is destroyed at some $S3 \in (S1,S2]$.)

Table 3: The "mutable objects" theory.

Definitions in Chunk Theory

CD.1 ghost$(C,S) \Leftrightarrow \neg$material$(C,S)$.
(Definition of ghost: A chunk is a ghost iff it is not material.)

CD.2 place$(C,S)$ = image(placement$(C,S)$, cshape$(C)$)
(Definition of place: The region occupied by $C$ in $S$ is the image of its shape under its placement.)

CD.3 sub_chunk$(C1,C2) \Leftrightarrow$
$\exists_S$ material$(C2,S) \wedge$ place$(C1,S) \subseteq$ place$(C2,S)$.
(Definition of sub-chunk: $C1$ is a sub-chunk of $C2$ iff $C1$ occupies a subset of $C2$ in some situation where $C2$ is material.)

CD.4 top_level$(C,S) \Leftrightarrow$
[material$(C,S) \wedge \forall_{C1}$ [material$(C1,S) \wedge$ sub_chunk$(C,C1)] \Rightarrow C1 = C$].
(A top-level chunk is a maximal material chunk relative to the sub-chunk relation.)

Axioms of Chunk Theory

CH.1 good_shape(cshape$(C)$).
(Chunks have a good shape.)

CH.2 [good_shape$(R1) \wedge R1 \subseteq$cshape$(C2)] \Rightarrow$
$\exists^1_{C1}$ $R1$=cshape$(C1) \wedge$ sub_chunk$(C1,C2)$.
(Every reasonably-shaped subregion of a chunk is a chunk.)

CH.3 continuous(placement$(C)$, $S$).
(The placement of chunk $C$ is continuous in every situation.)

CH.4 [sub_chunk$(C1,C2) \wedge$ material$(C2,S)] \Rightarrow$ material$(C1,S)$.
(A sub-chunk of a material chunk is itself material.)

CH.5 [sub_chunk$(C1,C2) \wedge$ material$(C2,S)] \Rightarrow$
placement$(C1,S)$ = placement$(C2,S)$.
(A sub-chunk of a material chunk has the same placement.)

CH.6 material$(C,S) \Rightarrow \exists_{C1}$ top_level$(C1,S) \wedge$ sub_chunk$(C,C1)$.
(Every material chunk is a sub-chunk of a top-level chunk (possibly itself).)

CH.7. [material$(C1,S1) \wedge$ ghost$(C1,S2)] \Rightarrow$
[$S1 < S2 \wedge$
$\exists_{S3,C2}$ $S1 < S3 \leq S2 \wedge \neg$sub_chunk$(C1,C2) \wedge$ top_level$(C2,S3) \wedge$
intersect(place$(C1,S3)$, place$(C2,S3)$)].
(A material chunk $C1$ can only turn into a ghost if its interior is penetrated by a top-level chunk.)

CH.8 [top_level$(C1,S) \wedge$ top_level$(C2,S) \wedge C1 \neq C2] \Rightarrow$
$\neg$intersect(place$(C1,S)$, place$(C2,S)$).
(Two top-level chunks cannot intersect.)

Table 4: Chunk Theory

### Inferences

The model supports exact predictions: given the positions and shapes of all the objects at the start of a time interval, and given the motions of all the objects throughout the interval, predict the identity and shapes of the final objects at the end of the interval. This is the kind of prediction that is carried out in CAM machining programs (Ji and Marefat 1997).

It also supports kinematic inferences of other kinds. For example, (Davis 1993) gives the proofs of the following statements:

- A blade that starts outside the target cannot carve a purely internal cavity inside the target.
- If a convex blade is restricted to linear motions, then carving out a $k$-face convex polyhedron requires at least $k$ separate cutting operations.

In our original scenario 2, of the cookie dough, this model supports most of the inferences one would want to make about cutting the dough with cookie cutters, assuming that the dough is otherwise rigid during the cutting process. For instance, one can conclude that, if the dough is cut in the center by a cutter that is a simple, non-closed, curve, then no cookie has been separated out. One can conclude that, if the horizontal projections of two cuts with ordinary cutters overlap, then the cookies cut out are the connected components of the intersection and the set differences of the two regions within the cutters.

### Observations

The strongest aspects of this formalization are, first, its generality, the fact that slicing, stabbing, and filing can all be treated together; and, second, its clarity; potential confusions are almost entirely resolved. If you try just to write down everything you know about "cutting", you are apt to find that there are a large number of issues to resolve, and that it is difficult to ensure that you are resolving them all consistently. This approach takes care of all these.

Moreover, these models seem cognitively plausible as far as they go. It seem very natural to think about individuated objects being gradually shaved away by a cutting process; it seems almost as natural to think about chunks of material, particularly when the extent of the object is either unknown, as in figure 7, or is very much larger than the region being operated on. The theories are certainly rather abstract and bloodless, mostly, I suspect, due to the absence of any dynamic theory. A lot of one's experience of cutting has to do with the forces and motions involved in sawing, stabbing, and so on, and these have all been abstracted away in this microworld.

## Advantages of microworlds

In this section and the next, we discuss the strengths and flaws of this approach of constructing microworlds to formulate a competence theory. Regrettably, the distinction between strengths and flaws is not as clear-cut as one might like. Some apparent strengths may actually be flaws; some apparent flaws may actually be just hard problems that would be encountered in any methodology.

The first and foremost advantage of the competence theory approach is that it takes us away from the painfully vague problem, "Is concept / distinction / fact $X$ part of naive physics?" and replaces it by the much more hard-edged, pragmatic, bottom-line, engineering-type question, "Is $X$ useful over a given class of inference?" For instance,

- Is an elastic collision between solid objects an instantaneous event, involving an instant change in velocity, or is it a prolonged process, involving an extended period of contact, a continuous change in velocity, and a deformation of the objects involved?

- Can a physical object be truly a point, or a curve, or a surface?

It is difficult to justify or even to assign meaning to a claim that one or the other of these is the "true" naive view. It is much easier to say that one or the other is an adequate model over a given class of inferences. Discussions such as that mentioned above about whether a painting on the wall is in the room can be avoided. What is actually going on, geometrically and physically, is quite clear enough and easily described. How you choose to define "in", whether you want to define the spatial extent of the "room" to include the walls, and whether you want to define the "walls" to include the painting (is the painting *part of* the wall or merely attached to it?) are comparatively unimportant and arbitrary decisions about the symbols "in", "room" and "wall".

This freedom from worrying about whether concepts are truly naive comes about primarily because, while Hayes' project requires that naive conclusions be drawn from naive premises, our project require only that naive conclusions be derivable; the premises need not be formulated in naive terms. Therefore, whereas Hayes' project requires that every concept be examined for its true naivity, and rejected if it is not genuinely naive, for us it suffices to have a large collection of naive conclusions. To carry out our project, in other words, it suffices to be able to generate a large collection of inferences that are unquestionably commonsensical; we never have to decide of a given inference that it is *not* commonsensical.

The problem of finding an appropriate level of generality, which we considered above, is likewise considerably clarified in the new approach. To attain maximal inferential power, one always goes to the highest level of generality that has any justification within the scope of the microworld. For example, in the cookie-cutter example, one can derive the rule from a very general theory of volume of regions together with the physical rule that the volume of the dough remains nearly constant while being rolled out. This general theory of volume will serve for many other inferences that involve reshaping of malleable, incompressible material, so it is advantageous to do this at a general level. On the other hand, there is probably nothing to be gained from abstracting further to the general notion of a Lebesgue integral in a general measure space; within commonsense physics, there will be no interesting generalizations to be obtained from this more abstract notion.

Once we are using microworlds in a competence theory, it becomes almost irresistibly tempting to consider competence over particularly interesting limited classes of inferences as a final goals in themselves. One can therefore contemplate the possibility of using multiple, mutually inconsistent, microworlds for the same phenomena, depending on the scope of inferences being considered and the precision required. For instance, there are many different theories that describe solid objects with varying degrees of accuracy: pure kinematics, quasi-statics, Newtonian dynamics of rigid objects, elastic solid objects, and so on. Each of these theories is useful under suitable circumstances. This is more difficult to justify in the project of expressing "naive physics", where we are presumably looking for a coherent universal theory.

This ability to consider microworlds for limited purposes has a number of advantages. First, it makes the analysis much easier; we can focus in on getting some particular class of inferences to work without worrying how these will fit with all the rest of naive physics. Second, it allows much closer ties to practical applications. Most practical AI physical reasoning programs work within a quite limited scope. For instance, many of the programs that do mechanical reasoning (Joskowicz and Sacks 1991), (Faltings 1987) work within the microworld of solid object kinematics or some small extension of it. As we shall argue further below, this tie to practical applications is very valuable for a number of reasons. Third, as the work on automated modelling (Nayak 1994) has shown, there can be considerable computational advantage to being able to choose, for a given problem, models of the correct level of precision and detail, so that correct answers can be reached without unnecessary excess computation. The study of alternative microworlds connects directly to this kind of study.

Focussing on the model rather than the axiomatization has the usual advantages of making it much easier to ensure consistency and to avoid unintended consequences. As discussed earlier, a concrete extensional model is necessary consistent and precisely defined, and so avoids much of the conceptual inconsistency and incoherence that can arise in the axiomatic approach.

## Dangers and difficulties of microworlds

This revised approach does not, however, take us out of the woods and cure all of our methodological difficulties. On the contrary, though some difficulties are alleviated from Hayes' original formulation, many are no lighter, and some are worse.

The chief problems are these:

- Commonsense reasoning is not an autonomous task domain.
- It is hard to find natural sources for commonsense inferences in a single microworld.
- The number of potential microworlds is vast, and the methodology provides no guidance for choosing between them.
- The focus on microworlds rather than axioms encourages (a) excessive specificity; (b) overemphasis on mathematical abstraction and elegance; (c) overemphasis on deductive reasoning.
- There is no easy way to extend or integrate microworlds.
- The method involves a great deal of hairsplitting of essentially vacuous issues.

We will elaborate on each of these individually.

### Not a task domain

The central objective in the new approach is to develop a competence theory for commonsense physical reasoning. But a competence theory must describe competence in some particular task, and "commonsense reasoning" is not, in itself, a task.[11] That is to say, it is not a cognitive activity that takes place by itself in people, or that would be of any value taking place by itself in a computer; it is an aspect of other cognitive tasks, such as planning actions, natural language understanding, expert systems, and so on. Moreover, the connection to commonsense reasoning is the most poorly understood aspect of these tasks, and at the current stage of understanding, such systems are very rarely improved by any attempt to incorporate commonsense reasoning.

Commonsense inference is thus an ill-understood module of a much larger task. It is therefore very difficult to be sure what the inputs and outputs of this module should be; that is, to decide how a commonsense inference should be formulated in order to serve the purposes of these larger tasks. In considering commonsense inference for a natural language processor, for example, it is difficult to know which aspects of the inference are part of the purely linguistic component and which parts are

[11]It is noteworthy that in the paradigmatic case of a competence theory, natural language syntax, the Chomskian linguists have felt obliged to focus on a very narrow and artificial task, that of judging grammaticality, rather than think about more ecologically valid tasks, such as producing or comprehending natural language. It might be worth considering whether some analogous task could be found in our domain.

part of the commonsense reasoner. It is also difficult to know what is involved in "understanding" a given text.

For instance, consider the text, "Use a rolling pin to roll out the cookie dough on a flat surface that has been covered with flour. Then cut it into pieces with a cookie cutter." Interpreting this text involves making the inference that "it" refers to the dough rather than the rolling pin, the surface, or the flour. This inference requires a combination of linguistic rules and commonsense reasoning. But it is not easy to tell what commonsense inference, precisely, is involved here. Do we want to infer that it is difficult to cut a rolling pin, or that it is unusual to do so, or that doing so will serve no purpose in the recipe? In the same way, it is difficult to know what is needed to achieve "understanding" of the text. Does the task of natural language understanding, as such, require inferring that the surface is horizontal? or that the cutter is moved downward through the dough to the surface? (Translation of a text into another language often does require this kind of knowledge, in order to choose the proper spatial terms.) In short, the problems of what the representation of a text should be and of how world knowledge can be used in linguistic analysis are very obscure, and therefore it is difficult to get guidance for commonsense reasoning and representation from linguistic examples. ((Bloom et al. 1996) is a fascinating survey of work in cognitive science on the connection between language and spatial reasoning.)

Similar ambiguities appear in relating commonsense reasoning to robotics. Here they take the form of the uncertainty of knowing what a high-level plan looks like, and how it relates to low-level robot programming. Suppose we want to build a robotic system that can carry out the cookie dough plan. Then the system effectively infers the statement "If program $P$ is carried out on robot $R$ in situation $S$, then the goal of having cookies will probably be achieved." This is not, in itself, a statement analyzable within a commonsense physical theory, as $P$ mostly consists of a lot of low-level robot-specific instructions governing manipulation, vision, and hand-eye coordination. It is not at all clear what high-level plan should be the subject of commonsense reasoning, or what statements should be inferred about such a plan. The general issue here is the problem of determining what issues should be addressed in high-level planning and what is the form of a high-level plan. Again, these difficulties make it hard to use robotic programming as a guide to commonsense reasoning.

Let me clarify the problem here by contrasting commonsense reasoning with two other hard tasks. Automatic dictation, from voice to manuscript, is hard, but at least we know the form of the input (an acoustic string) and the output (a sequence of characters) and we have an unlimited collection of examples where we know that a correctly working program will produce output $O$ for input $I$. Fluid flow analysis for rocket testing is a hard module to build, but again we know that the input should be the boundary conditions for the relevant PDE and a specification of the desired precision, and that the output needed is a field of fluid flow of that precision. The difficulty with commonsense reasoning is that there are very few instances where we can be really sure what the input and the output should be.

That recent research in knowledge representation tends to suffer from its disconnection from to practical research has been argued by numerous researchers, including Morgenstern (1997a) and Etherington (1997).

## No natural sources for single microworlds

Once we have chosen a microworld, we have to find a collection of inferences within that microworld as a testbed. It is important that the collection should well represent the range of commonsense inferences in the domain, in terms of the physical phenomena considered, the types of partial knowledge, and the directions of inference. If the collection of inferences is too narrow, then it is likely that the model developed from them will be too weak or the language too inexpressive.

The problem then is, how does one assemble a suitably broad collection of commonsensical inferences within a given microworld? The best way would be to choose a task that is easily carried out by naive subjects, such as vision or language interpretation, and collect the commonsensical inferences within this microworld involved in that task.[12] But this is hard to do, as discussed in the previous section. Reasoning for expert systems, or processing of specialized natural language text, or planning for special-purpose robotics often stays largely within a small microworld, but rarely covers the range of commonsense inference; these tend to be confined to a few types of inference (e.g. prediction) and to very few types of partial knowledge. Within these confines, they go far beyond commonsense reasoning in specialized techniques and knowledge (otherwise, they wouldn't be *expert* systems.) Natural language processing of general text and planning in rich environments uses many more types of inference, but only occasionally do these fall within the chosen microworld.

The method of exploring variants, advanced in section 1, often yields a collection of interesting problems, but it has a number of built-in biases: it tends to favor prediction problems over other directions of inference; it tends to favor fairly complete specifications; and, being generated by the researcher herself or sympathetic colleagues, it can easily be biased toward conforming to the theory the researcher has in mind. Also, a researcher who has thought for a long time about a given microworld may well tend to exaggerate how easily naive subjects can make certain inferences, so she may include as commonsensical inferences that are in fact quite difficult.

For example, suppose we want to evaluate how well the model and language for cutting solid objects presented above characterize commonsense inference in that domain. How can we go about such an evaluation?

A claim of adequacy must be that a significant fraction of the commonsense inferences in this domain can be justified in this theory. How do we find or define a space of typical commonsense inferences within this microworld? We can look at the inferences that a CAM machining program is implicitly carrying out, or the additional inferences that it would be useful for such a program to carry out. But most of these are of the form "To create hole $H$ in object $O$, move cutter $C$ through path $P$", which can all be satisfied by a substantially simpler model of cutting, such as one in which each operation with the cutter is taken to be atomic; and by a simpler language, such as one in which all geometric descriptions are exact. Most of the other inferences used in the CAM program fall outside the microworld, such as restrictions on the thinness of the parts that can cut out of a given material with a given cutter. We can look at natural language processing of a technical text describing machining. This will probably yield a slightly broader class of inferences within the microworld than the CAM program, but still a quite restricted one. We can look at unrestricted text, but how frequently does any interesting issue in cutting solid objects arise in novels or in the newspaper?

So the question, which is naturally often raised, of how this theory could be implemented, is one that I can hardly answer, because I have no idea what such an implementation would be supposed to *do*. I could implement a predictive program that takes exact initial shape descriptions and description of motions and output final shape descriptions, but this has been done by the CAM people much better than I could do it. I could set a general-purpose complete theorem prover on the axiom set of tables 3 and 4, together with a set of temporal and geometric axioms, but for a theory of this complexity, I would not expect an answer in reasonable time to any but the most trivial queries. What I am looking for is an inference engine that will work efficiently over the space of commonsensically obvious inferences, but I don't know what that space *is*, let alone how to design an inference engine for it.

I am not, of course, arguing that commonsense inference has no practical application. I am

[12]Interestingly, the original plan for CYC (Lenat et al. 1986) was to express the background knowledge needed to understand encyclopedia articles (hence the name); they later report that "that use of external written materials has become increasingly rare." (Lenat and Guha 1993).

arguing that the practical applications are apt to be few until we have gotten far past simple microworlds to very broad theories. A program that could do general commonsense reasoning would be of immense value; a program that could do physical commonsense reasoning, broadly interpreted, would be of very great value; but a program that can do commonsense reasoning about cutting solid objects, or similarly narrow domains, would be of very little value. Therefore, it is very difficult to know what any of these programs should do about cutting solid objects. We don't know what a program that only does commonsense reasoning about cutting solid objects should do, because there is almost nothing useful that it can do. We don't know what kinds of reasoning about cutting solid objects a general commonsense reasoning program would be called on to do because it will only very occasionally called upon to carry out an inference that is both non-trivial and lies entirely within this microworld.

This problem is serious, not just because the absence of short-term payback makes it difficult to attract the interest of colleagues, students, and funding agents, though these considerations are not to be sneezed at. Far more importantly, it means that there is almost no way to guide research in microworlds to evaluate what progress is being made, except for the judgment and taste of the researcher (McDermott 1987). We have to work almost blind until the work is almost complete.

## Innumerable microworlds

Hayes' project is large but, at least in principle, it is finite; once the knowledge of all naive physics has been formalized, the project is done. Our project, by contrast, is infinitely open-ended or nearly so; one can continue to make up and analyze new microworlds forever, by slightly varying the set of assumptions involved. For example, there is an endless collection of variations on the blocks world: Blocks may stack in towers one on one, or they may be rectangular of varying sizes, or they may have more general shapes; time and space may be continuous or discrete; there may be one hand or many hands, and, if many, they may work one at a time or concurrently and they may interact in any of several ways; and so on. This is useful for the teacher giving a class in KR who needs simple examples to assign, but for the researcher, only a few of these merit any study. The methodology described above does not give one any clue as to when the analysis of a new microworld is worthwhile. The choice of where to invest energy is left entirely up to the judgment of the researcher, and KR research has always been remarkably apt to leave the great ocean of truth undiscovered, while crowding around an empty Clorox bottle on the beach.

## Excessive specificity

A microworld is, so to speak, an entire alternative universe that approximates or abstracts the real one. In formulating a microworld, therefore, it is often difficult to avoid ontological overcommitment; being overly specific merely for the sake of having a well-defined model.

Suppose, for example, we want to describe cell division. At the beginning of the process, there is one cell; at the end, there are two cells. For simplicity, it is certainly easiest to say that up to a certain time there is one cell, called $A$, and after that time, there are two cells, called $B$ and $C$. Clearly, however, isolating the moment in the process that divides one from two is entirely arbitrary and pretty pointless.[13] So we would like to be agnostic about this. In the axiomatic approach, this agnosticism is very easily attained, through the following axioms:

1. At the beginning of cell division, $A$ exists.
2. At the end of cell division, $B$ and $C$ exist

[13] Deciding which state holds at that exact moment is doubly arbitrary and pointless.

3. At all times during cell division, exactly one of two possibilities holds: (1) $A$ exists; (2) $B$ and $C$ exist.
4. It cannot be that $B$ and $C$ exist at one time and then $A$ exists at a later time.,

By contrast, the whole spirit of the microworlds approach militates against this kind of agnosticism. Characteristically a microworlds approach will feel obliged to define a criterion for the temporal individuation of cells, and this criterion will impose a unique solution to the question of the dividing point. Without such a criterion, the only way to achieve conditions (3) and (4) is to define the temporal lifetime of one cell in terms of the lifetime of another, and this kind of recursion loses many of the advantages of microworlds, such as the easy guarantee of consistency. I'm not saying it can't be done; I'm saying that someone working within the microworlds methodology is much less likely to adopt such a solution or to be satisfied with it.

The model-based methodology also pushes toward excessive specificity and concreteness in the concepts considered. The focus is on concepts that are easily characterized in terms of their spatial/temporal/material aspects to the exclusion of more nebulous but important concepts attached to causality and teleology. Consider the following inference

> If you cut through an object anywhere near the center, you will probably destroy its functionality.

The inference is important, true, and commonsensically obvious, but is likely to be omitted in a model-based theory, because of the difficulty of defining "functionality". It is also unlikely to be found as a sample commonsense inference by the method of proposing variants, because it is too general.

## Excessive mathematization

Similarly, the model-based methodology leads to an excessive interest in constructing elegant and minimal mathematical models rather than expressive, messy models. For example, the kinematic theory of cutting solid objects presented earlier is elegant and simple, easily stated and formalized, covering a wide range of phenomena with a few rules.

The dynamic theory of cutting solid objects, by contrast, is complex, haphazard, and incomplete. Consider the range of motions, forces, and behaviors involved in slicing through butter, sawing wood, driving a nail, screwing a corkscrew, and drilling a hole. A model that characterizes all these fully at the commonsense level will necessarily involve a large number of separate rules and constraints governing these separate common cases. (The theory at the atomic level is simple, but there the structural representations needed to describe these various scenarios is very complicated.) Moreover, these rules and constraints are not disconnected arbitrary facts, but are deeply interconnected. For instance, anyone who has observed the processes of butter being sliced and of wood being sawed will expect, from the nature of the processes and the materials, that butter can be sliced more thinly that wood can be sawed. But it is not easy to find the general rules that give rise to that expectation.

The researcher who wants to move forward producing models will therefore tend to avoid this kind of microworld, as these models are, in every respect, harder to develop. The ontology and language are much richer; the theory is much more complex; it is hard to be sure that the various constraints and rules are mutually consistent; it is hard to be sure that all cases have been covered. Paradoxically, one suspects that this kind of model would also be harder to "sell" as legitimate research; they look like a mere translation of random obvious statements into formalese. In fact, the immense gap between a mere translation of random statements and a coherent theory is no smaller in a complex theory than in a simple one, but the coherence of the complex theory is harder to achieve, to convey, and to grasp.

In fact, as the microworlds become more complex, the need for complex systems of constraints on the models means that the distinction between the axiomatic approach and the model-based approach tends to vanish. Each of these constraints is, effectively, an axiom; the difficulties of dealing with the constraints are almost the same as the difficulties of dealing with a set of axioms; and the advantages of a model-based approach over an axiomatic approach, in terms of clarity and of easily-verified consistency are much diminished.

Having constructed elegant models for simple domains, the next temptation is to spend time proving neat theorems about them, or in them. These are often of doubtful relevance. A twenty-two page proof that two theories of cutting are mathematically equivalent (Davis 1993) certainly does not represent any cognitive activity that anyone (except myself) has ever carried out, nor any computational activity that any program is ever likely to carry out. Now, of course, I can and do justify such research in terms of the methodology itself: a program that can reason flexibly about cutting must be based on a good model of cutting; the two models potentially have different advantages as regards automated inference; if we want to use them both, we should understand the relation between them; hence, it is of value to know that they are in fact equivalent. Which is all very well, but all the same the gap between application and research has gotten rather large.

This mathematizing tendency also affects the formulation of queries. Previously, we suggested that the special rule "The thinner you roll the cookies dough, the more cookies you can cut out," could be deduced as a consequence of more general geometric rules, plus rules that the cookie dough has fixed volume, and that cutting out cookies corresponds to dividing the region of the dough into vertical cylinders with some fixed cross-section. But this "generalization" fails to capture the causal direction of the special rule, the fact that the baker can *choose* how thick to roll the dough and where to cut the cookies, and that these choices determine the number of cookies obtained. By contrast the geometric rules are atemporal; they would equally apply to a case where someone was assembling a mass of cookie dough out of cookie pieces, and where the choice of the number of cookie pieces would determine the eventual volume of cookie dough. A large part of mathematical training involves making this kind of abstraction automatic; it eventually becomes so much second nature that perceiving the distinction between the original rule and its abstraction requires a conscious effort.

## Too much stress on deduction

Being centered around semantic consequence, the microworlds approach tends to focus exclusively on deductive reasoning. It can, perhaps, be extended to types of plausible reasoning based on a strong semantic model, such as circumscription or probabilistic reasoning, but would be very difficult to integrate with such theories as default reasoning, reasoning by analogy, case-based reasoning, and so on.

## Extending and combining microworlds

An advantage to Hayes' project is that, as the aim at every step is always a complete theory of naive physics, and as every axiom of every cluster is "true"' relative to that overall theory, once you have correctly formulated an axiom or a cluster, you can count on it and keep it. If it is true, it remains true. In the microworlds approach, by contrast, a model that has been constructed to characterize a very narrow microworld does not usually apply in a broader world. Models, theories, languages, and axioms almost always require some revision in going from a narrower to a wider setting, and may well require complete reworking from scratch.

Let us first consider a case where the extension of one model to a richer model has a straightforward logical structure. The kinematic theory of rigid solid objects can be extended to a dynamic theory by adding mass, force, momentum, and so on, and imposing Newton's laws. This is what Giunchiglia and Walsh (1992) call a "theorem increasing" extension; the language is richer and the axioms of dynamics are a strict superset of those of kinematics. It is also, correspondingly, "model[14] decreasing" (Nayak and Levy 1994); if $H$ is a history consistent with the dynamic theory then the "projection" of $H$ obtained by eliminating all aspects of the history except shape and position is consistent with the kinematic theory.[15]

A more complex example is the extension of the kinematic theory of rigid solid objects (KRSO) to the theory of cutting rigid solid objects (CSO) described above. This is a model-increasing extension; any history consistent with the KRSO theory is also consistent with CSO. Correspondingly, it is a theorem-decreasing extension. This seems a little odd, as the CSO contains all kinds of axioms and inferences about cutting that don't apply in KRSO, but actually these are all vacuously true in the KRSO case. For instance, it is true in KRSO that if a knife cuts through an apple, the apple will be split into two parts, because the antecedent of the implication is necessarily false. (Note that statements of feasibility like "You can cut through an apple with a knife," are not part of CSO, as we have defined it.)

However, the simple characterization above requires a significant qualification. Starting with CSO, it is easy to construct KRSO as a special case by adding to the "mutable objects" theory the axiom that the shape of an object is constant; or adding to the "chunk" theory the axiom that all chunks are eternal. Going to KRSO to CSO, which is the more likely order of development, is much more difficult. In the natural logical statement of KRSO, shown in tables 5-7, there is no need for the fluent "material($O$)", since all objects are eternal, and the function "shape($O$)" maps an object $O$ to a spatial region rather than to a fluent, since the shape of an object is fixed. Thus, developing the mutable object theory of CSO from KRSO requires significant reworkings of the conceptualization, ontology, and language in addition to changing the axioms. (Note that only one of the axioms from table 7 survives unchanged in table 3.) Developing chunk theory requires even greater ontological changes, though, curiously, fewer axiomatic changes (three axioms from table 7 appear in the same form in table 4).

[14] This is "model" in the strict metalogical sense.

[15] If you allow the imposition of arbitrary external forces and impulses as boundary conditions then a version of converse also holds: Given any (piecewise twice-differentiable) motion satisfying the kinematic constraints, there is some way of imposing external forces so that, in the dynamic theory, the objects execute the specified motion. At this point, the question of which, if either, direction is "theorem decreasing" and "model increasing" becomes rather murky.

| Sort | Letter |
|---|---|
| Spatial regions (set of points) | $R$ |
| Rigid mappings | $M$ |
| Temporal situations | $S$ |
| Fluents | $F$ |
| Objects | $O$ |

Table 5: Logical Sorts in KRSO

Temporal:
  value_in$(S, F)$ — Function. Value of fluent $F$ in situation $S$.

Spatial:
  intersect$(R1, R2)$ — Predicate. Region $R1$ intersects $R2$.
  good_shape$(R)$ — Predicate. Region $R$ is non-empty, open, bounded, connected, and equal to the interior of its closure.
  image$(M, R)$ — Function. The image of region $R$ under mapping $M$.
  continuous$(F, S)$ — Predicate. $F$ is a continuous function of time at situation $S$. $F$ is a fluent whose value in each situation is a rigid mapping.

Physical: Primitive Symbols
  placement$(Q)$ — Function. The fluent of the mapping from the shape of $Q$ to the place of $Q$.
  shape$(O)$ — Function. The point set occupied by $O$ in a standard orientation.

Physical: Defined Symbols
  place$(Q)$ — Function. The fluent of the region occupied by $Q$ in situation $S$.

Table 6: Non-logical primitives in KRSO

K.1 place$(O, S)$ = image(placement$(O, S)$, shape$(O)$).
(Definition of place: The region occupied by $O$ in $S$ is the image of its shape under its placement.)

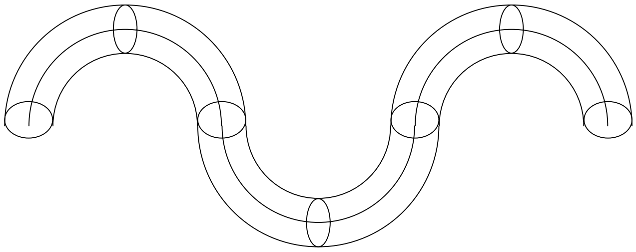
K.2 $O1 \neq O2 \Rightarrow \neg$intersect(place$(O1, S)$, place$(O2, S)$).
(Two objects do not overlap.)

K.3 good_shape(shape$(O)$). (Every object has a good shape.)

K.4 continuous(placement$(O)$,$S$).
(The placement of object $O$ is continuous in any situation $S$.)

Table 7: Axioms of KRSO

Similar difficulties are encountered in trying to combine two microworlds; all too often, one finds that each microworld depends on assumptions that are violated in the other. Let me discuss an example that has been fretting me for some years. I have a theory of cutting rigid solid objects. I also have a theory of strings, presented briefly in (Davis 1995). The form of this theory is determined by the following considerations:



The central curve is the core of the string, and the small circles are cross-sections.

Figure 8: Theory of string

A. The length of a string is constant.

B. Strings are very flexible.

C. It is tempting to make strings one-dimensional curves, but that creates difficulties. For instance, if two strings touch one another, or one part of a string touches another part, then, if the strings are truly one-dimensional, it becomes very difficult to specify which string is on which side. Consequently, it becomes difficult to fix the rules so that one string cannot pass through the other. It is much easier to specify a reasonable physics if strings are required to be fully three-dimensional objects, though thin.

D. The diameter of a string is generally much less than its length, and the shape of its cross section is unimportant for most purposes.

E. We wish to abstract away the details of the composition of the string, which varies from one string to the next, and focus on the external characteristics, which are very much the same from one string to another.

To accommodate these constraints, I proposed the following kinematic theory of strings and solid objects (Figure 8):

> A string is characterized by its length $L$ and its radius $R$. At any given moment, the *core* of the string lies on a curve $C$ of arc-length $L$. The cross-section of the string perpendicular to the core is a circle of radius $R$; that is, the extension of the string occupies all points of the form $\mathbf{q} + \Delta\hat{N}$ where $\mathbf{q}$ is a point in the core $C$; $\hat{N}$ is normal to the curve $C$ at $\mathbf{q}$; and $\Delta \leq R$. The string observes the following constraints:
>
> - The string moves continuously.
> - The string does not overlap any solid rigid object.
> - The string does not overlap any other string.
> - The string does not overlap itself. That is, there cannot be two distinct points $\mathbf{q}_1$ and $\mathbf{q}_2$ on curve $C$; two normals $\hat{N}_1$ and $\hat{N}_2$ to $C$ at $\mathbf{q}_1$ and $\mathbf{q}_2$; and two quantities $\Delta_1$, $\Delta_2 < R$, such that $\mathbf{q}_1 + \Delta_1\hat{N}_1 = \mathbf{q}_2 + \Delta_2\hat{N}_2$

This theory is reasonably straightforward, and integrates directly with the kinematic theory of rigid objects. It supports inferences such as, "If string A is looped, with one end flush against the other, and string B is likewise looped, and the two cores are topologically linked, then the two strings cannot be separated from one another while keeping them both looped." The topological part of this proof is not easy, but the physics is simple.

The problem now is, how can the theory of strings be combined with the theory of cutting? The difficulty is that halfway through the process of cutting the string, the string has a notch that has been vaporized out of it. The theory of strings, as stated above, assumes that a string has a circular cross-section everywhere.

Now there may be a good, or at least a deep, reason for this difficulty. The model of string as a uniform tube is an abstraction of many different string-like substances: woven string, braids, single fibers, metal wires, rubber-coated wires, even linked chains. The abstraction is reasonable across a wide range of behaviors, but it falls apart in scenarios that probe the internal structure of the string. (By definition, of course: a scenario that distinguishes one internal structure from another is precisely one in which the internal structure cannot be abstracted away.) Chief among these is cutting or partially cutting the string; what happens when you cut halfway through a string is quite variable, depending on what the string is made of. Hence, it is not surprising that modelling cutting string is not a simple extension of modelling string.

On the other hand, cutting string is not, after all, a very esoteric activity, and the fact that, when you cut a string, you end up with two shorter strings is one of the best-known and most important properties of string. Three related reactions to the above difficulty come to mind immediately. The first is that we don't care what's going on in the middle of cutting string; all we care about is the end result. The second is that we don't generally care about strings that have been halfway cut through; when we start to cut a string, we usually complete the job. The third is that the requirement that strings have a circular cross-section should be dropped; there are many strings, including sneaker laces and strings that have been partially cut through, that do not.

The first of these reactions is actually a fallacy, based on the ease with which human reasoners solve and therefore ignore the frame problem. After all, cutting string does not create a physics-free zone, and we would care very much if string, while it was being cut, spat forth a poison that was fatal on contact. So the reaction "we don't care" is presupposing some very strong constraints on the behavior of the string while being cut that carry over from before it was cut, and our problem is precisely to state these constraints in a way that integrates with the rest of our theory.

The second reaction is more productive. We could look for a model in which the string is never partially divided, by positing that the string splits in two as soon as it is penetrated by the blade. This can be accommodated in chunk theory by observing that, unlike soap or marble where any reasonable subset can be carved out, strings can really only be cut straight through. (If you do manage to cut a string lengthwise, then what you get may very well not be a string.) Therefore, if we take a "chunk" to be "something that can potentially be cut out of the material," then the chunks in the string are precisely lengthwise segments of the string. If we apply our rule from chunk theory that a chunk vanishes as soon as it is penetrated, then what we get is precisely the above model, that the string is split as soon as the blade enters it. (Chunk theory also allows a more elegant expression of the rule that the string does not overlap itself.)

This theory seems elegant enough, and it does the right thing for almost all cases of cutting string, so in that sense it is a reasonable competence theory.[16] Unlike the microworlds we have looked at before, however, the description here is never either true or plausible; strings do not split in two the instant that the knife enters them, and one does not imagine that they do. Moreover, on the rare occasions when it is obvious that the knife will partially cut the string but not wholly, this gives a prediction that is neither right nor plausible.

[16] I have not worked through this theory carefully, and so there may be some technical problems that arise. It is a little worrisome, for instance, that in this theory a solid object exists over a time interval that is closed on the left and open on the right, while a string exists over an interval that is open on the left and closed on the right. My guess, though, is that this does not raise any real difficulties.

What we have done, in short, is to construct a concrete model of the process of cutting, which has the correct starting and ending behavior for completed cuts and the correct interaction during cutting with the rest of the world (i.e. none). Then this model will do the right thing as long as we never have to reason about incomplete cuts or about the state of the string during cutting. The fact that it is easier to construct such a overly specified model rather than just characterize correctly the starting and ending states and the interaction with the rest of the world is a fine example of how the model-based methodology pressures one into overly specific models.

The third suggestion, that we should allow strings with non-circular cross-sections, has in its favor that it is true and it will have to be accommodated in an ultimate commonsense theory. However, the theory of non-cylindrical strings is significant more complicated than the theory of cylindrical strings, for a number of reasons. First, non-cylindrical strings can twist; in cylindrical strings, twist is invisible and can therefore be ignored. Second. non-cylindrical strings are more restricted in the shapes they can attain. For instance, it is not possible to wind a sneaker lace tightly in the plane of the lace itself without buckling, because the outer diameter of the lace becomes so much longer than the inner diameter. The microworlds approach has value insofar as it allows us to focus on a natural class of issues, and it would seem natural that we should be able to reason about the very common and familiar process of cutting string without getting involved in all the rare and specialized issues of oddly-shaped string.

## Hairsplitting

By this stage of the paper, few readers will need more illustrations of this point! A little earlier, we were patting ourselves on the back because we could avoid two-hour discussions on the meaning of "in", but though that particular vacuous argument is avoided, many others come in to take its place. The kind of precision needed in this kind of analysis seems to require inescapably that all kinds of borderline cases and anomalies be resolved.

In the case of real borderline cases — Is a platypus a mammal? Is glass a solid? What is an impulse? — this is somewhat tolerable, as scientists and engineers who study this kind of issue also spend serious work doing this kind of resolving of borderline cases. Even here, one's intuition is that human commonsense reasoning is distinguished by its willingness to admit the existence of borderline cases, and its non-insistence on tying all these down; and one would like the theory of automated commonsense reasoning to be similarly flexible. What is truly intolerable, however, is the amount of time and effort that must be spent in resolving purely hypothetical and imaginary borderline cases and anomalies, just for the sake of having clear-cut definitions and models — When you turn on a light, is it on or off at the exact dividing moment? Do objects occupy open or closed regions in space? What happens if an object is sliced simultaneously by infinitely many blades? No scientist or engineer would dream of wasting her time in this way; here we are in company only with mathematicians and philosophers. Mathematicians have it comparatively easy; the hairs only have to be split when choosing definitions, not when proving theorems; mathematics tends to have few definitions and many theorems; and hairs can be split along any lines that seem most convenient. By contrast, we spend much more of our time defining concepts and models, and we are under pressure to make our definitions more or less fit with commonsense concepts. Philosophers have it even worse than we do; rather than analyzing straightforward concepts like cutting string, they are trying to deal with Truth, Justice, and Beauty. On the other hand, of course, the reward for their efforts is a better understanding of Truth, Justice, and Beauty, whereas the best we can hope for is a better understanding of how to formalize cutting string.

## The role of microworlds in the larger scheme of things

When we are all done — when we have encoded all of commonsense physical knowledge in a declarative knowledge base, and implemented all commonsense physical reasoning in an inference engine over the knowledge base — how will our work on microworlds be reflected in the final product? Three posssibilities come to mind.

One possibility is that we will attain Hayes' dream of a single consistent theory that incorporates all commonsense physical knowledge and supports all commonsense physical inferences. In this case, our microworlds would certainly serve no intrinsic logical function. They would survive at most as organizational structures, clusters within the knowledge base supporting efficient retrieval. More likely, in view of our observations above of their characteristic inextensibility, they would simply vanish. Their whole function in the research project, then, would have been as stepping stones and training exercises for the eventual theory. Certainly, there would be no point in ever going backwards; once a more comprehensive microworld had been satisfactorily formulated, there would be no interest in considering special cases.

A second possibility, along the lines of (Addanki et al. 1989), is that the final knowledge base would be structured entirely in terms of mutually inconsistent microworlds, with no overall theory. There would be rules at the meta-level for choosing the microworld suitable to a given problem, or for resolving conflicts when different microworlds gave different answers, but there would no object-level theory that would serve as the final court of appeals in such cases.

A third possibility combines the two of these. There is a structure of microworlds, integrated through meta-rules, but at the top of this structure is a single Hayesian theory to which all questions can be ultimately referred (Figure 9.) The microworlds approximate the overall theory, and are computationally more tractable. This can be viewed as a special case of the second structure, in which there happens to be a single overarching top-level microworld. Alternatively, it can be viewed as an instance of the first structure, by taking the overall theory to be logically primary, and viewing calculations involving the microworlds as approximation heuristics to the overall theory.

Undoubtedly, we at present know much too little to predict which, if any, of these will win out. However, a few pros and cons may be observed.

The first structure, of a single comprehensive theory, is certainly the simplest from a logical standpoint. Indeed, the idea of using multiple worlds runs seriously counter to goals of a declarative representation or a knowledge-based analysis. This becomes particularly evident in cases where you want to use two conflicting models in a single problem, either to describe two different objects in the problem, or two different times, two different places, two different scales of granularity, or two different interactions. For instance, to calculate the tides, you first calculate the motions of the earth and the moon around the sun as if they were point objects; then treat the earth as a solid of very complex shape with bodies of water. The reasoner must somehow keeping the inferences from each microworld within the range of its applicability and avoid making inferences from these microworlds that make nonsense of the problem being solved; and this idea of a limited range of inference is not one that works easily within the standard view of a knowledge-based system.

This difficulty is not necessarily alleviated by formally incorporating a microworlds structure inside a single first-order theory, as is done in the theory of "contexts" (McCarthy 1993). The fundamental issue is not so much whether there is one first-order theory or a system of many; it is whether the knowledge base as a whole can be considered as a declarative, transparent representation of a coherent body of knowledge. If the structure of contexts can be given a clear extensional significance justifying the axioms that connect them, that would be a great gain. But an obscure and arbitrary first-order theory of contexts has no advantage over an obscure and arbitrary system of procedural techniques for combining alternative theories.

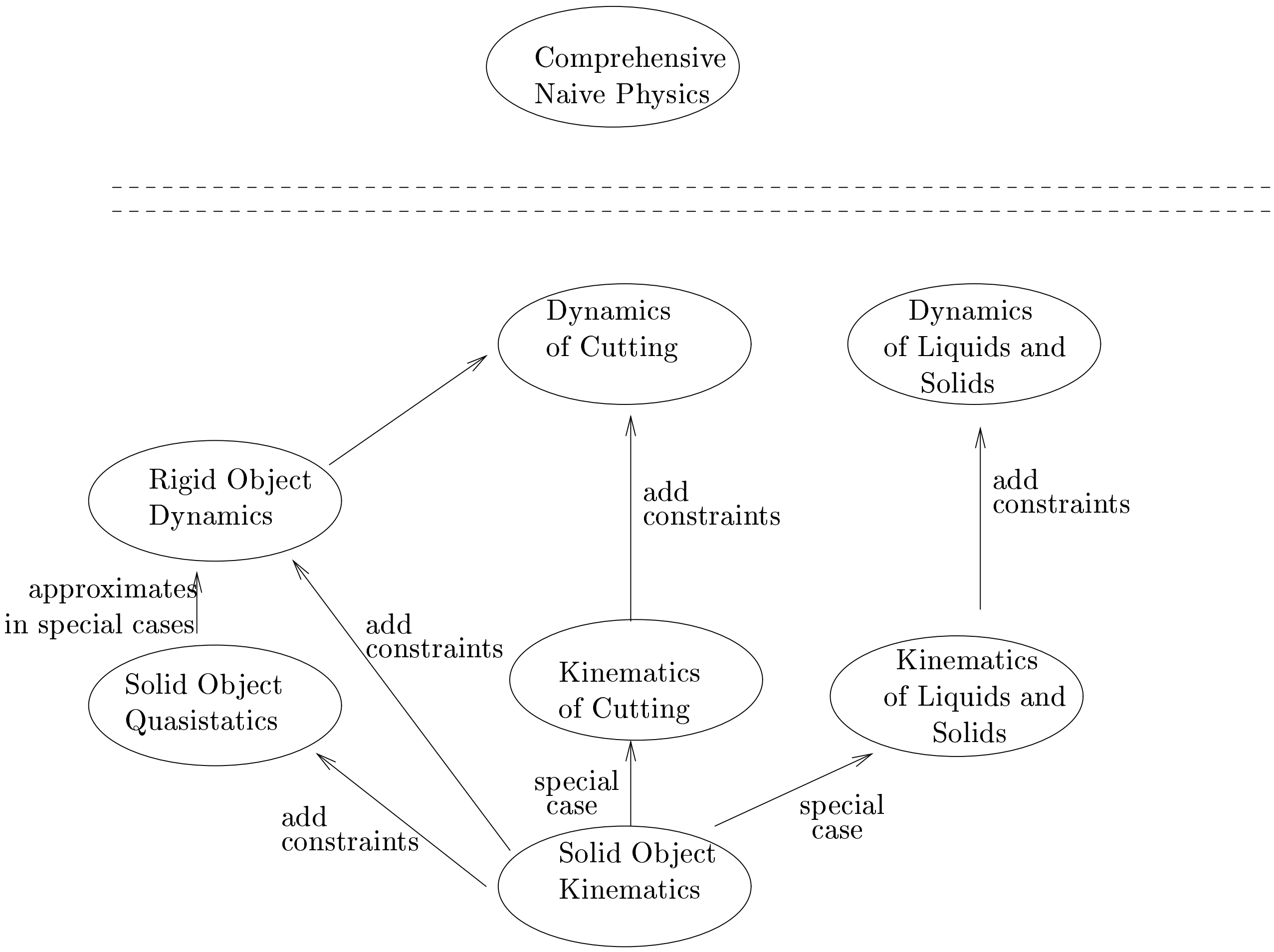



Figure 9: Part of a structure of microworlds

Moreover, I have not found any convincing arguments that a structure of alternative microworlds is a particularly plausible cognitive model of commonsense physical reasoning. I do not know of any cases where commonsense reasoning seems to require the combination of two conflicting models. I suspect that in developing a commonsense physical reasoner, our ultimate aim should be something like Hayes' uniform, comprehensive theory. Therefore we should tend to stress the steady expansion of the scope and detail of our theories, rather than pursue such virtues as simplicity or tractability.

By contrast, in developing an automated reasoner for expert scientific or engineering reasoning, the idea of a structure of alternative microworlds approximating a single ultimate correct theory seems much more promising. In formulating and solving a problem, a scientist/engineer will almost always simplify, abstract, and approximate; she can generally describe the approximations she is making and, to some extent, explain why they will simplify the problem, and why she expects that the answer will still be useful. A large part of scientific and engineering training has to do with learning a library of useful approximations and abstractions and learning how to apply these to different problems. Indeed, there is a very active research area trying to develop an account of the relations between microworlds and to see how a structure of microworlds can be used in automated expert physical reasoning.

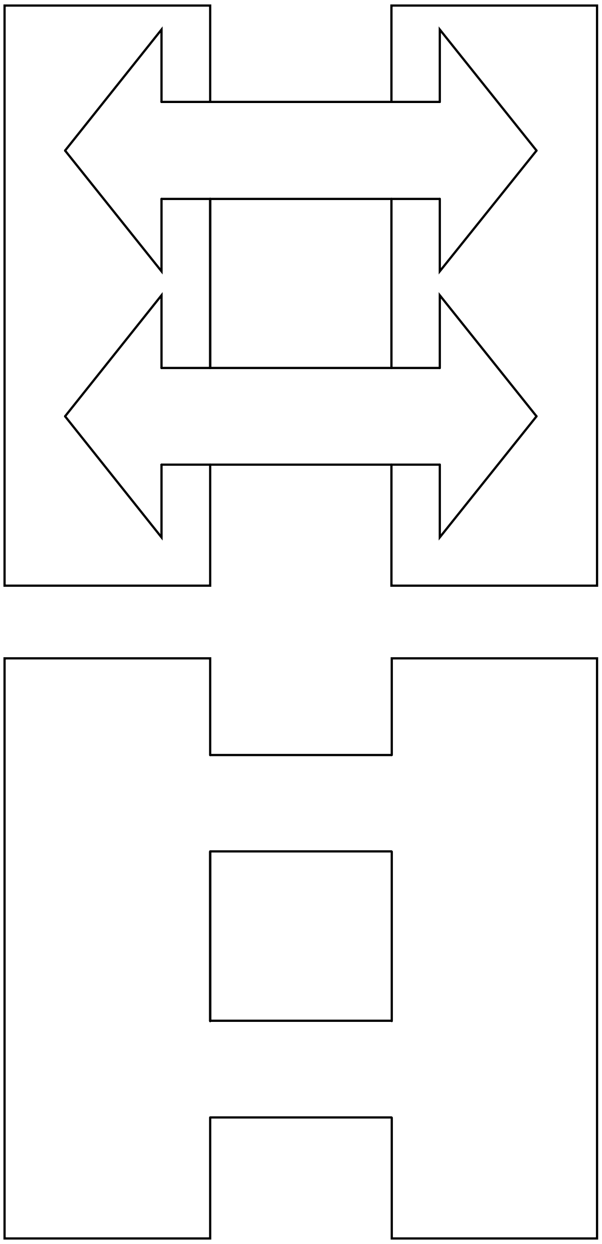
I am not convinced, however, that microworlds as such play a major role in this kind of reasoning. Approximation and abstraction in physical reasoning takes many forms: Objects of complex characteristics may be approximated by objects of simpler characteristics (e.g. a real resistor by an ideal resistor); a group of objects may be approximated by a single object (figure 10); a group of simple objects may be approximated by a single complex object (e.g. a chain of rigid links by a string). The case where a complex microworld is approximated by a simpler microworld is just one of many cases, and it is not obvious that it is a particularly important special case. Also, it is a mistake to assume, as is sometimes done (e.g. (Weld 1992)) that a problem is easier to solve in a formally simpler theory than in a more complex theory; in many important cases, the reverse holds. For example, solid object dynamics is a formally simpler theory if friction is excluded than if it is included, but in many problems, such as the system in figure 11, prediction is easy in a theory with friction — the system remains static — but difficult in a frictionless theory.

Since many problems in physical reasoning can be solved within the scope of a single microworld, the development of microworlds remains useful in developing automated expert physical reasoners. I suspect, however, that the study of the relationships *between* microworlds and the manipulation of microworld assumptions will be much less important in developing sophististicated reasoning techniques.

## Conclusions

Despite all these difficulties and objections, and despite the increasing impatience of the AI community with laboriously hand-coded knowledge-based systems (e.g. (Charniak 1993, preface)), I find our original scenarios — the staked plant, the cookie dough, the baby bottles, and a myriad similar situations — too fascinating and compelling to abandon. I still feel that it is wise to begin by developing representations for a knowledge-level analysis, and that the method of microworlds is the most promising approach that we have. The main task now, therefore, is to develop more and richer microworlds.

As we have discussed, we can expect the next generation of microworlds will be more difficult in every respect than those we have already seen. If we look at microworlds such as the dynamics of cutting, we expect to find that microworlds will be more complex and narrower; that reasoning will rely more on plausible inference; that the spatial component of reasoning will be both more complex and less clearly defined; and that immediate connections to useful applications will become fewer.



The four objects on top can be treated as if they were the single object below.

Figure 10: Abstraction of structure: Several objects become a single object

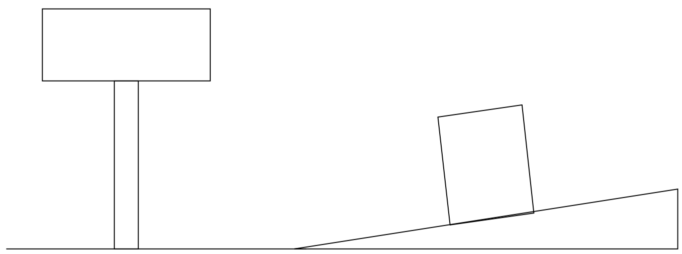


Figure 11: A simple system with friction

But if we have patience enough to stick with it, we should eventually have a remarkable theory.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