

A First-Order Theory of Stanislavskian Scene Analysis: Appendix A (Object-level Proof)

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1 Appendix Overview

This is the appendix to the paper “A First-Order Theory of Stanislavskian Scene Analysis.” It contains an extended sketch of an object-level proof that a particular scene analysis — of a small sample textbook scene, used to teach principles of acting and directing — is *coherent*, as it was defined in the paper.

The proof is simple in structure, and consists mostly of matching the definitions with the scene analysis and domain axioms. It is useful in that it gives a detailed example of what a scene analysis looks like, and illustrates how one would prove that a scene analysis is coherent. As is typical in exercises of this sort, it was also useful in uncovering errors, omissions, and typos in earlier versions of the axioms and definitions.

The small sample script used is analogous to the toy problems commonly used by AI researchers. We make simplifying assumptions of various sorts:

1. The script is clearly much smaller than the typical script that actors and directors use.
2. A comprehensive scene analysis, even for such a small script, would distinguish with finer granularity among the different dramatic actions. For example, we have repeated instances of dramatic actions such as *request*, *refuse*, and *taunt*. A finer-grained analysis might distinguish, e.g., between *requesting*, *insisting*, and *threatening*. Incorporating this range of dramatic actions in any serious fashion would necessitate a comprehensive analysis of speech acts as well as an analysis of the subtle differences between closely related dramatic actions. This is not within the scope of this paper, although such an analysis would enrich future work in this area.
3. In this scene analysis, there is a one-to-one relationship between lines and locutionary actions, and between locutionary actions and dramatic actions. This is in general not the case. Frequently, one locutionary act — even a short one — can map to several dramatic actions; while one dramatic action can be spread over several locutionary actions.
4. The scene analysis is set up so that one action follows another without gaps, agents only perform one action at a time, and agents’ actions do not overlap. Clearly these restrictions would not be possible in more realistic scene analyses.
5. Because the focus of the paper is on the representation of dramatic actions, there is no representation at this stage for locutionary actions: each locutionary action is simply represented as $Do(a, utter(string))$.¹

¹It might be conjectured that we should at least use a basic division of locutionary actions into declarative sentences, questions, imperatives, and so on. But even this is a non-trivial task in a domain in which the affect and context can

6. The domain axioms for break-ups, relationships, and the like are clearly ad hoc. The research agenda for this paper focusses on scene analysis, not on these other domains; and, as is customary in formal commonsense research, we allow ad-hoc representations for domains outside of the research focus.
7. Similarly, we do not have a comprehensive treatment of many of the temporal issues that arise. For example, to express one of our domain axioms correctly would require a notion of indirect causation. There has been some preliminary work on this notion [1], but it remains a hard problem, and we simply posit a predicate to capture the notion. Likewise, we have ignored a problem that comes up with respect to the integration of branching and linear time: the prediction of what *will* happen in a branching model of time in which multiple histories, or paths through the branching time tree, are feasible. There are a variety of approaches to deal with this difficulty, including the approach, taken by [3], of considering only those histories that are well-behaved in some sense defined relative to the context and domain. It is because we punt on these and similar issues that we are calling this an extended proof sketch rather than a complete proof.

Notes on Syntactic Sugar:

We will use the following syntactic sugaring conventions:

- $Do(a, Act(\vec{x}) \mid P(\vec{x}))$ denotes the action of a doing actional Act with the range restricted to \vec{x} in the obvious way.
 - In general, $Occurs(Do(a, Act(\vec{x}) \mid P(\vec{x}), s1, s2) \Leftrightarrow \exists x (P\vec{x} \wedge Occurs(Do(a, Act(\vec{x}, s1, s2)))$.
 - We will use $\neg f$ to mean the “negation” of a fluent f in the following sense: For each fluent f , we can posit a fluent f' such that $Holds(s, f) \Leftrightarrow \neg Holds(s, f')$. We can write f' as $\neg f$.

2 The Sample Script and Scene Analysis

2.1 Sample Scene

The sample scene presented in the paper was adapted from Stanley Kahan and Kenneth W. Rugg, *Introduction to Acting*, 1998:

- (1) A: Give me that.
- (2) B: No.
- (3) A: Give it to me.
- (4) B: I don't think so.
- (5) A: Come on: I really want it.
- (6) B: No!
- (7) (A grabs it from B.)
- (8) B: Well?
- (9) A: Well what?
- (10) B: Well, say something.
- (11) A: What do you want me to say?
- (12) B: You might have something to say.
- (13) A: I'm not going to say anything.

2.2 Definition of Scene and Scene Analysis

(This subsection is repeated from the paper for convenience.)

affect the categorization of the locutionary act. For example, when a character utters the word “Well?”, he may be asking a question, issuing a command, or even making a statement.

We define a scene SC as a tuple $\langle Char, \Sigma \rangle$, where $Char$ is the set of agents/characters in the scene and Σ is a sequence of (mostly) locutionary actions. (Σ may include dramatic actions that are entailed by the script. However, in general, most dramatic actions are introduced during the SAP.)

We define a scene analysis $SA(SC, A')$ as a tuple

$\langle Char, \Sigma, [SS, SE], BStory(A', SS), Obj, \Delta(A', SS, SE), \Pi \rangle$, where

- $Char$ and Σ are as above,
- SS and SE are the starting and ending situations of this instantiation of the scene,
- $BStory(A', SS)$ is the backstory of character A' up to situation SS , defined as a set of sentences of the form $Holds(s, f)$ or $Occurs(ac, s1, s2)$, where $s \leq SS$ and $s2 \leq SS$ and a is in $Char$.
- Obj is a set of fluents, the objectives of A' ,
- Δ is the dramatic history of the scene, defined as a set of sentences of the form $Holds(s, f)$, $Occurs(a, s1, s2)$, $Executes(a, strat, f, s1, s2)$, or $StartExecute(a, strat, f, s1, s2)$ where $SS \leq s, s1, s2 \leq SE$, and a is in $Char$.
- Π relates subsets of Δ to subsets of Σ . That is, Π associates dramatic actions with lines in the script. In general, one line of the script may be associated with several dramatic actions, and one dramatic action may be associated with several lines in the script, even if interleaved with other character's locutionary actions. (An example of the latter is the conversation-cum-side-by-side-monologue between Lenny and Curley's wife, in Steinbeck's "Of Mice and Men," directly before Lenny accidentally suffocates Curley's wife.)

Note that we can define the predicate $StartExecute(a, strat, f, s1, s2)$ in terms of the fluent $CStrat(a, f, strat)$, denoting that $strat$ is a current strategy of a in achieving f . In particular, Then $StartExecute(a, strat, f, s1, s2)$ is true iff for any s in the interval $[s1, s2]$, $Holds(s, CStrat(a, f, strat))$, if $[s1, s2]$ be the earliest interval in which $CStrat(a, f, strat)$ holds. Because of this equivalence, we will when convenient use $CStrat$ statements to characterize scene analyses rather than $StartExecute$ statements.

Let $\Gamma(SA(SC, A'))$ be the union of the sentences in the backstory and the dramatic history.

2.3 Scene and Scene Analysis for Sample Scene

2.3.1 Scene

For the sample scene,

$Char = \{A, B\}$, the two characters in the scene.

Σ is the sequence of mostly locutionary actions, along with dramatic actions entailed by the script: these can include dramatic actions that are in the stage directions (e.g., the *smothers her* of *Othello's* Act 5, Scene 2, where Othello kills Desdemona) or actions that are entailed by an utterance of the character. In this case, there is one dramatic action in the stage directions, that of A grabbing the object from B. Thus in this case, Σ is the following sequence, which consists basically of a rewriting of the script.

$Do(A, utter("Give me that. "))$
 $Do(B, utter("No. "))$
 $Do(A, utter("Give it to me. "))$
 $Do(B, utter("I don't think so. "))$
 $Do(A, utter("Come on; I really want it. "))$
 $Do(B, utter("No. "))$
 $Do(A, grabfrom(B, o) \mid Object(o))$
 $Do(B, utter("Well? "))$
 $Do(A, utter("Well what? "))$
 $Do(B, utter("Well, say something. "))$
 $Do(A, utter("What do you want me to say? "))$
 $Do(B, utter("You might have something to say. "))$
 $Do(A, utter("I'm not going to say anything. "))$

2.3.2 Scene Analysis

For the sample scene, *Char* and Σ are as above.

The starting and ending situations are *S1* and *S15*.

Backstory

$BStory(B, S1)$, the backstory, consists of a set of sentences that describe the situation at the beginning of the scene, or that tell of events that happened prior to the start of the scene. In this case, the backstory says that B and A are in a relationship, that B does not currently love A, that B is a non-confrontational sort of person, that B currently possesses a one-way travel ticket for B to travel to the Bahamas at a future date, and that B believes that nobody believe that B wants to get out of the relationship.

The representation of these sentences, respectively, is:

Backstory Premise 1 $Holds(S1, InRelationship(B,A))$

Backstory Premise 2 $\neg Holds(S1, Loves(B,A))$

Backstory Premise 3 $\forall s Holds(s, Nonconfrontational(B))$

Backstory Premise 4 $\exists date, sd, se (sd > se \wedge Holds(S1, Possess(B, TravelTicket1)) \wedge DateOf(sd, date) \wedge travelinfo(TravelTicket1, B, OneWay, Bahamas, date))$

Backstory Premise 5 $\forall s2 B(B,S1,s2) \Rightarrow$
 $\neg \forall x, s3 B \neq x \Rightarrow ((B(x, s2, s3) \Rightarrow Holds(s3, desire(B, \neg InRelationship(B,A))))))$

The term *InRelationship* will frequently be abbreviated as *InRel*.

Objectives

Obj, the (set of) objectives of B, is the fluent *HadEasyBreakup(B,A)*, indicating that (when the fluent holds) B has had an easy breakup from A.

Dramatic History

Δ , the dramatic history of the scene, is the following set of sentences:

DH 1 $StartExecute(B, StratRunaway(B,A), HadEasyBreakup(B,A), S1, S8)$

DH 2 $StartExecute(B, StratHide(B, desire(B, \neg InRel(B,A)), UnknownByOthers(B, desire(B, \neg InRel(B,A))), S1, S8)$

DH 3 $Occurs(Do(A, request(B, give(A, TravelTicket1))), S1, S2)$

DH 4 $Occurs(Do(B, refuse(give(A, TravelTicket1))), S2, S3)$

DH 5 $Occurs(Do(A, request(B, give(A, TravelTicket1))), S3, S4)$

DH 6 $Occurs(Do(B, refuse(give(A, TravelTicket1))), S4, S5)$

DH 7 $Occurs(Do(A, request(B, give(A, TravelTicket1))), S5, S6)$

DH 8 $Occurs(Do(B, refuse(give(A, TravelTicket1))), S6, S7)$

DH 9 $Occurs(Do(A, grabfrom(B, TravelTicket1)), S7, S8)$

DH 10 $Occurs(Do(A, read(TravelTicket1)), S8, S9)$

DH 11 $StartExecute(B, StratTaunt, HadEasyBreakup(B,A), S9, S14)$

DH 12 $Occurs(Do(B, taunt(A, desire(B, \neg InRel(B, A))))), S9, 10)$

DH 13 $Occurs(Do(A, refusebait(B, taunt(A, desire(B, \neg InRel(B, A))))), S10, S11)$

DH 14 $Occurs(Do(B, taunt(A, desire(B, \neg InRel(B, A))))), S11, S12)$

DH 15 $Occurs(Do(A, refusebait(B, taunt(A, desire(B, \neg InRel(B, A))))), S12, S13)$

DH 16 $Occurs(Do(B, taunt(A, desire(B, \neg InRel(B, A))))), S13, S14)$

DH 17 $Occurs(Do(A, refusebait(B, taunt(A, desire(B, \neg InRel(B, A))))), S14, S15)$

DH 1 and 2 say that B starts to execute the Runaway and Hide strategies between S1, the start of the scene history, and S8. DH 11 says that B starts to execute the Taunt strategy between S10 and S14. The remaining axioms state the dramatic actions that occur in the scene. A's actions are asking for the travel ticket (DH 3, 5, 7), grabbing it (DH 9), reading it (DH 10), and refusing to take the bait of B's taunting (DH 13, 15, 17). B's actions are refusing to give A the travel ticket (DH 4, 6, 8), and, once A has already grabbed the travel ticket, taunting A (DH 12, 14, 16).

Given the equivalence between *StartExecute* and *CStrat*, we get from DH 1, 2, and 11, the following:

DH 18 $\forall s (s \in [S1, S8] \Rightarrow Holds(s, CStrat(B, StratRunaway(B, A), HadEasyBreakup(B, A))))$

DH 19 $\forall s (s \in [S1, S8] \Rightarrow Holds(s, CStrat(B, StratHide(desire(B, \neg InRel(B, A))), UnknownByOthers(B, desire(B, \neg InRel(B, A))))))$

DH 20 $\forall s (s \in [S10, S14] \Rightarrow Holds(s, CStrat(B, StratTaunt(A, desire(B, \neg InRel(B, A))), HadEasyBreakup(B, A))))$

Mapping between dramatic history and scene

Π , the relation mapping subsets of the dramatic history to subsets of the scene, is extremely simple in this example. It relates one dramatic action to each line of the script and strategies to several lines of the script as explained above. Note that one dramatic action (reading the ticket) is not explicit in the screen. By convention, dramatic actions that do not occur in the script will be mapped to the empty set, subscripted by the surrounding lines of script. Thus we have:

$\Pi =$

$\{ (\{1\}, \{ DH 3 \}),$
 $(\{2\}, \{ DH 4 \}),$
 $(\{3\}, \{ DH 5 \}),$
 $(\{4\}, \{ 6 \}),$
 $(\{5\}, \{ DH 7 \}),$
 $(\{6\}, \{ DH 8 \}),$
 $(\{7\}, \{ DH 9 \}),$
 $(\{ (S9, S10) \}, \{ DH 10 \}),$
 $(\{8\}, \{ DH 12 \}),$
 $(\{9\}, \{ DH 13 \}),$
 $(\{10\}, \{ DH 14 \}),$
 $(\{11\}, \{ DH 15 \}),$
 $(\{12\}, \{ DH 16 \}),$
 $(\{13\}, \{ DH 17 \}),$
 $(\{ (1), (2), (3), (4), (5), (6) \}, \{ DH 1 \}),$
 $\{ (1), (2), (3), (4), (5), (6) \}, \{ DH 2 \},$
 $\{ (9), (10), (11), (12), (13), (14) \}, \{ DH 11 \} \}$

3 Strategies and Domain Axioms

3.1 General axioms and definitions on strategies

These are taken, with some modification and corrections, from the paper.

We say that a scene analysis is *coherent* if the following conditions hold:

- [1] The scene objectives are motivated with respect to the backstory
- [2] Any other objectives arise from the original scene objectives, the strategies taken to pursue objectives, and the facts that are true during the scene
- [3] An agent's actions during the scene follow from his objectives and chosen strategies
- [4] An agent will not continue a strategy that he believes has failed.

Let SC be a scene and $SA(SC, A')$ a scene analysis for character A' , as defined above. Let $\Gamma(SA(SC, A'))$ be the set of wffs associated with the scene analysis, as defined above. Let $\Gamma(CSK)$ be a set of sentences representing a body of commonsense knowledge. (E.g., this might include commonsense domain theories about wooing spouses, and fathers' reactions to their daughters' elopements.)

Then

Definition 1 SA is coherent iff $\Gamma(SA(SC, A')) \cup \Gamma(CSK) \models$

1. (motivation of scene objectives)
 $(\forall s \in [ss, se] \text{ Holds}(s, SObj(A', f)) \Rightarrow \text{Holds}(s, \text{Motivated}(A', f))) \wedge$
2. (subgeneration of other objectives)
 $(\text{Holds}(s, CObj(A', f)) \Rightarrow \text{Holds}(s, SObj(A', f))) \vee$
 $\exists \text{ strat}, ac (\text{Holds}(s, CStrat(A', f, strat)) \wedge \text{ActionOf}(ac, strat) \wedge \neg \text{Holds}(s, \text{Done}(ac, A', strat))) \wedge$
 $(\text{Precond}(f, ac) \vee \text{FailCond}(\neg f, ac))) \wedge$
3. (strategy pursuit only for objectives, and only if not failed)
 $(\text{Holds}(s, CStrat(A', f, strat)) \Rightarrow$
 $((\text{Holds}(s, CObj(A', f)) \wedge \text{StrategyFor}(strat, f)) \vee (\text{Holds}(s, CObj(A', f')) \wedge \text{StrategyFor}(strat', f') \wedge \text{StratPart}(strat, strat'))$
 \wedge
 $\neg B(A', s, s') \Rightarrow \text{Holds}(s', \text{StrategyFailed}(A', f, strat))) \wedge$
4. (actions are performed by A' only if done as part of some strategy and only if it is believed that they will not fail)
 $(\text{occurs}(s, s', do(A', act)) \Rightarrow$
 $\text{Holds}(s, CStrat(A', f, strat)) \wedge \text{Holds}(s, \text{PotAct}(A', act, strat)))$
 $\wedge \neg B(A', s, s') \Rightarrow$
 $\forall f (\text{FailCond}(f, Do(A', act)) \Rightarrow \text{Holds}(s', f))$

Definition 2 $\text{Holds}(s, \text{Done}(ac, a, strat)) \Leftrightarrow$

- $$\text{ActionOf}(ac, strat) \wedge$$
- $$\exists f, ss', ss, sa, sb \text{ Holds}([ss, s], CStrat(a, f, strat)) \wedge$$
- $$\forall s' ss' \leq s' < ss \Rightarrow \neg \text{Holds}(s', CStrat(a, f, strat)) \wedge$$
- $$sa \geq ss \wedge sb \leq s \wedge \text{Occurs}(ac, sa, sb)$$

An actional *act* is a *potential action* for an agent a pursuing some strategy *strat* if all the precursors of *ac* in *strat* have already been done:

Definition 3 $\text{Holds}(s, \text{PotAct}(a, act, strat)) \Leftrightarrow$

- $$\forall ac \text{ Precursor}(ac, Do(a, act), strat) \Rightarrow \text{Holds}(s, \text{Done}(ac, a, strat))$$

3.2 Domain-specific descriptions of and axioms on strategies

Strategy 1 $\text{StratRunaway}(a1, a2)$:

- $$\text{Occurs}(Do(a1, \text{hide}(\text{desire}(a1, \neg \text{InRel}(a1, a2))))), s1, s2) \wedge$$

$$\begin{aligned} & \text{Occurs}(\text{Do}(a1, \text{taketrip}(\text{OneWay}, \text{dest})) \mid \text{Remoteloc}(\text{dest}, s2, s3) \wedge \\ & \text{Occurs}(\text{Do}(a1, \text{writeletter}(a2, \text{let})) \mid \text{contentof}(\text{let}, \text{desire}(a1, \neg \text{InRel}(a1, a2))), s4, s5) \wedge \\ & s3 < s4 \end{aligned}$$

Strategy 2 *StratHide*(a1, f):

$$\begin{aligned} & \text{Occurs}(\text{Do}(a2, \text{request}(a1, \text{act})), s1, s2) \wedge \\ & \text{Occurs}(\text{Do}(a1, \text{act}), s2, s3) \wedge \text{LeadTo}(s3, s4) \\ & \exists a3 \\ & (a3 \neq a1 \\ & \quad \forall s5 (B(a3, s4, s5) \Rightarrow \text{Holds}(f, s5)) \\ & \Rightarrow \\ & \exists s6 \text{Occurs}(\text{Do}(a1, \text{refuse}(\text{act})), s2, s6) \end{aligned}$$

Strategy 3 *StratTaunt*(a1, a2, f):

$$\begin{aligned} & \neg \text{Occurs}(\text{do}(a2, \text{takebait}(\text{taunt}(a2, f))), s3, s4) \\ & \wedge \neg \text{RepeatOccurs}(\text{Do}(a1, \text{taunt}(a2, f)), 3, s1, s4) \\ & \Rightarrow \text{Occurs}(\text{Do}(a1, \text{taunt}(a2, f)), s5, s6) \\ & s2 < s3 \wedge s4 < s5 \end{aligned}$$

Strategy description 1 describes the runaway strategy. It consists of three actions. First, a1 must hide his/her desire to break off the relationship with a2. This continues right up until the time that a1 takes a one-way trip to a remote destination. Sometime after that, a1 writes a letter to a2, informing a2 of a1's desire to end the relationship. Note that this strategy refers to another strategy, Hide.

Strategy description 2 describes the the hide strategy. As with many strategy descriptions, it can be incomplete. This strategy description merely says that if a1 is trying to hide something, and a2 asks a1 to do some action, and doing that action would entail that at some future point, there is some person (other than a1) who would come to know that thing, then a1 will refuse to do the action.

Strategy description 3 describes the taunt strategy. This strategy consists of taunting someone up to three times unless they have already taken the bait.

We have the following axioms on strategies:

Strategy 4 *StrategyFor*(*HadEasyBreakup*(a1, a2), *StratRunaway*(a1, a2))

Strategy 5 *StrategyFor*(*HadEasyBreakup*(a1, a2), *StratTaunt*(a1, a2, *desire*(a1, \neg *InRelationship*(a1, a2)))

Strategy axioms 4 and 5 say that two strategies for having an “easy” (non-confrontational) breakup are running away from and taunting. (A richer theory would rank these strategies according to desirability.)

Strategy 6
$$\begin{aligned} & \text{Occurs}(\text{Do}(a2, \text{request}(a1, \text{act})), s1, s2) \wedge \\ & \text{Occurs}(\text{Do}(a1, \text{act}), s2, s3) \wedge \text{LeadTo}(s3, s4) \wedge \\ & \exists a3 \\ & (a3 \neq a1 \wedge \\ & \quad \forall s5 (B(a3, s4, s5) \Rightarrow \text{Holds}(f, s5)) \\ & \Rightarrow \\ & \text{Holds}(s2, \text{ActionOf}(\text{StratHide}(a1, f), \text{Do}(a1, \text{refuse}(\text{act})))) \end{aligned}$$

Strategy 7 $\forall s \text{Holds}(s, \text{ActionOf}(\text{StratTaunt}(a1, a2, f), \text{Do}(a1, \text{taunt}(a2, f))))$

Strategy axioms 6 and 7 give actions that are part of their respective strategies. (These are the only axioms giving actions for strategies that we will need for the proof.) Strategy axiom 6 says that in a situation where one has been asked to do something that will entail revelation of what one is trying to hide, the strategy's action is to refuse to do the action. Strategy axiom 7 simply says that the taunt action is always a part of the taunt strategy.

Strategy 8 $(a1 \neq a3 \wedge (B(a3,s,s') \Rightarrow Holds(s',f)))$
 $\Leftrightarrow Holds(s, StrategyFailed(a1, UnknownByOthers(a1,f), stratHide(a1,f)))$

Strategy 9 $Holds(s, StrategyFailed(a1, UnknownByOthers(a1, desire(a1, \neg InRelationship(a1,a2)), Hide(a1, desire(a1, InRelationship(a1,a2))))))$
 $\Rightarrow Holds(s, StrategyFailed(a1, HadEasyBreakup(a1,a2), StratRunaway(a1,a2)))$

Strategy axiom 8 says that the hiding strategy fails iff the fact that one is trying to hide becomes known to others. Strategy axiom 9 says that a1's strategy to run away from a2 fails if his/her strategy to hide his/her desire to not be in a relationship with a2 fails. Note a major simplification in the theory as it has thus far been developed: There is no requirement that a strategy be currently or have been previously active in order for it to fail.

Strategy 10 $StratPart(StratHide(a1, desire(a1, \neg InRel(a1,a2))), StratRunaway(a1,a2))$

Strategy axiom 10 says that the hiding strategy is a part of the runaway strategy.

3.3 Domain Axioms

3.3.1 Axioms on Belief

Belief Axiom 1 *We assume the standard principles of a logic of belief as in [2] consequential closure, positive introspection, negative introspection, belief of all logical axioms, consistency, arrogance, etc..*

Belief Axiom 2 *If p is an axiom (even a domain axiom), then all agents believe p .*

Agents believe all relevant facts about the scenes: their scene objectives, current objectives, strategies, actions, etc. (However, they usually don't know all the objectives of other agents.)

Belief Axiom 3 $Holds(s1, SObj(a,f)) \Rightarrow \forall s2 (B(a,s1,s2) \Rightarrow Holds(s2, SObj(a,f)))$
 $Holds(s1, CObj(a,f)) \Rightarrow \forall s2 (B(a,s1,s2) \Rightarrow Holds(s2, CObj(a,f)))$
 $Holds(s1, CStrat(a,f, strat)) \Rightarrow \forall s2 (B(a,s1,s2) \Rightarrow Holds(s2, CStrat(a,f, strat)))$
 $Holds(s1, CAction(strat, a, f, ac)) \Rightarrow \forall s2 (B(a,s1,s2) \Rightarrow Holds(s2, CAction(strat, a, f, ac)))$
etc.

Agents are always aware of the actions they have performed:

Belief Axiom 4 $Occurs(do(a, act), s1, s2) \Rightarrow$
 $\forall s3 B(a, s2, s3) \Rightarrow$
 $\exists s4 occurs(do(a, act), s4, s3)$

This is a slight simplification; see [2] for a more thorough treatment of this issue. (Knowledge accessibility links should not be permitted to cross in the time structure.)

There are certain actions which, when performed, an agent is aware of, even if he is not the performing agent. These include actions which involve the agent. For purposes of this proof, we state this axiom for actions such as grabbing, refusing bait, taking the bait, and so on.

Belief Axiom 5 $act = do(a1, taunt(a2, f)) \vee act = do(a1, grabfrom(a2, o)) \vee act = do(a1, req(a2, act))$
 $\vee act = do(a1, ref(a2, act)) \vee act = do(a1, takebait(a2, f)) \vee act = do(a1, refusebait(a2, f))$
 $\wedge occurs(act, s1, s2) \Rightarrow$
 $\forall s3 B(a, s2, s3) \Rightarrow \exists s4 Occurs(act, s4, s3)$

We have one axiom on how beliefs and strategies interact. We say that if one strategy is part of another, and an agent happens to be executing both of them at some time, then if one fails, the other fails too.

$$\begin{aligned}
& \mathbf{Belief\ Axiom\ 6} \quad \text{StratPart}(\text{strat2}, \text{strat1}) \wedge \text{StartExecute}(a, \text{strat1}, f1, s1, s2) \wedge \text{StartExecute}(a, \text{strat2}, f2, s3, s4) \\
& \wedge s1 \leq s3 \wedge d4 \leq s2 \wedge s \in [s3, s4] \\
& \Rightarrow \\
& (\forall s' B(a, s, s') \Rightarrow \text{Holds}(s', \text{StrategyFailed}(a, f1, \text{strat1}))) \\
& \Leftrightarrow \\
& (\forall s' B(a, s, s') \Rightarrow \text{Holds}(s', \text{StrategyFailed}(a, f2, \text{strat2})))
\end{aligned}$$

3.3.2 Explanation Closure Axioms

We use explanation closure axioms, as in [4], to handle the frame problem. We will need to show that if an objective is motivated for an agent at some time, it stays motivated until some action happens that could result in his reaching his objective. We will also need to show that if an agent doesn't know that his significant other wants to break up with him, he won't know unless he is informed of it, or reads a letter telling him, or reads a travel ticket informing him that his significant other is planning a long, one-way trip.

$$\begin{aligned}
& \mathbf{Explanation\ Closure\ 1} \quad \text{Holds}(s1, \text{Motivated}(a1, \text{HadEasyBreakup}(a1, a2))) \\
& \wedge \neg \text{Holds}(s2, \text{Motivated}(a1, \text{HadEasyBreakup}(a1, a2))) \\
& (\text{Occurs}(\text{Do}(a1, \text{writeletter}(a1, \text{let})), s1, s2) \wedge \text{contentof}(\text{letter}, \text{desire}(a1, \neg \text{InRel}(a1, a2)))) \vee \\
& (\text{Occurs}(\text{Do}(a2, \text{takebait}(\text{taunt}(a1, \text{desire}(a1, \neg \text{InRel}(a1, a2)))))), s1, s2))
\end{aligned}$$

$$\begin{aligned}
& \mathbf{Explanation\ Closure\ 2} \quad \neg B(a2, s1, s1') \Rightarrow \text{Holds}(s1', \text{desire}(a1, \neg \text{InRel}(a1, a2))) \\
& B(a2, s2, s2') \Rightarrow \text{Holds}(s2', \text{desire}(a1, \neg \text{InRel}(a1, a2))) \\
& \exists a3 \text{Occurs}(\text{Do}(a3, \text{inform}(a2, \text{desire}(a1, \neg \text{InRel}(a1, a2)))), s1, s2) \\
& \vee \exists \text{let} \wedge \text{contentof}(\text{letter}, \text{desire}(a1, \neg \text{InRel}(a1, a2))) \wedge \text{Occurs}(\text{Do}(a2, \text{read}(\text{let})), s1, s2) \\
& \vee \exists \text{travelticket}, \text{dest}, \text{date} \wedge \text{travelinfo}(\text{travelticket}, a1, \text{One-way}, \text{dest}, \text{date}) \wedge \text{Remoteloc}(\text{dest})
\end{aligned}$$

3.3.3 Other domain axioms

If one doesn't loves someone, one doesn't want to be in a relationship with that person.

$$\begin{aligned}
& \mathbf{Domain\ Axiom\ 1} \quad \text{Holds}(s, \text{InRel}(a1, a2)) \wedge \neg \text{Holds}(s, \text{Loves}(a1, a2)) \\
& \Rightarrow \text{Holds}(s, \text{desire}(a1, \neg \text{InRel}(a1, a2)))
\end{aligned}$$

If one is non-confrontational, and desires not to be in a relationship with someone, then he is motivated to be in a state where he has had an easy breakup from that person.

$$\begin{aligned}
& \mathbf{Domain\ Axiom\ 2} \quad (\text{Holds}(s, \text{Nonconfrontational}(a1)) \wedge \\
& \text{Holds}(s, \text{desire}(a1, \neg \text{InRel}(a1, a2)))) \\
& \text{Holds}(s, \text{Motivated}(a1, \text{HadEasyBreakup}(a1, a2)))
\end{aligned}$$

There are no precursors in the hide strategy for the refuse action.

$$\mathbf{Domain\ Axiom\ 3} \quad \forall a1, f, \text{act2} \neg \exists ac1 \text{Precursor}(ac1, \text{refuse}(\text{act2}), \text{StratHide}(a1, f))$$

There are no precursors in the taunt strategy for taunting.

$$\mathbf{Domain\ Axiom\ 4} \quad \forall a1, f, \text{act2} \neg \exists ac1 \text{Precursor}(ac1, \text{taunt}(a2, f), \text{StratTaunt}(a1, a2, f))$$

Only one action can happen at a time.

Domain Axiom 5 $(Occurs(ac1,s1,s2) \wedge Occurs(ac2,s3,s4) \wedge overlap([s1,s2], [s3,s4])) \Rightarrow ac1 = ac2.$

A situation where one has physical possession of a travel ticket leads to a situation where one knows the information on that travel ticket.

Domain Axiom 6 $LeadTo(PhysPossess(a, travelticket), KnowInfor(a, travelticket))$

And: If a1 gives an object to a2, or a2 grabs an object from a1, a2 will physically possess that object at the end of the action.

4 Proof

We now prove that the scene analysis presented in Section 2.3.2 is coherent according to Definition 1. There are 4 clauses in the definition; we go through each in turn.

Clause 1:

The scene objective is $HadEasyBreakup(B,A)$. To prove Clause 1, we must therefore show that $\forall s (s \in [S1,S15] \Rightarrow Holds(s, Motivated(B, HadEasyBreakup(B,A))))$.

We first need to show that $Holds(S1, Motivated(B, HadEasyBreakup(B,A)))$. From Backstory premises 1 and 2 and Axiom 1, we get $Holds(S1, desire(B, \neg InRel(B,A)))$. In turn, this, together with Backstory premise 3 and Axiom 2 gives $Holds(S1, Motivated(B, HadEasyBreakup(B,A)))$.

We use Explanation Closure Axiom 1, along with the dramatic history of the scene analysis, and the restriction ?? that no more than one action happens at a time, to show that this holds not just in $S1$ but in all situations between $S1$ and $S15$.

This completes the proof of Clause 1.

Clause 2:

We need to show that all of B's current objectives are scene objectives or are generated from the scene objectives. There is only one scene objective in this example (DH1); thus this is trivially satisfied.

Clause 3:

We have to show that at any situation s , B's current strategy lines up with her current objective and that she does not know that it has failed. We must consider two cases.

Case 1: s in $[S1,S8]$. By DH1 and DH2, B starts executing the hiding strategy as part of the runaway strategy. This lines up with her current objective by Strategy Axioms 4 and 10. Moreover, we can show that from $s1$ to $s8$, B does not believe her strategy has failed. We can show this as follows: By Background Premise 6, in $S1$, nobody except for B knows that she does not want to be in a relationship with A. Using the action sequence given in DH3 through DH8, the restriction on only one action happening at a time, and the explanation closure axioms, we (and B) can infer that through $S8$, it is still the case that no one knows that B does not want to be in a relationship with A.

At $S9$, however, B's hiding strategy has failed. This follows from Strategy Axiom $s9$, DH10, and Belief Axiom 6.

The argument for Case 2 — s in $[S9, S15]$ — is entirely analogous. Thus, Clause 3 is satisfied.

Clause 4

We have to show that whenever B does an action, it is performed as part of her current strategy, and that she does not believe that the action will fail. That is, she does not believe that failure conditions hold for any of the fluents she is trying to achieve through her actions.

We go through each of B's 6 actions. These are given in DH4, DH6, DH8, DH12, DH14, and DH16. By DH2 and Strategy Axiom 6, the actions in DH4, DH6, and DH8 are clearly part of B's current strategy from $S1$ through $S8$. Moreover, there are no failure conditions for the refuse action, so the latter conjunct is trivially satisfied. (Note that this does not say that the general strategy will

be successful, just that one can always refuse to do something.) Analogously, by DH 11 and Strategy Axiom 5, the actions in DH12, DH 14, and DH16 are clearly part of B's current strategy from S9 onward. Again, there are not failure conditions for the taunting action, so the latter conjunct is trivially satisfied. (Again, this does not guarantee that the strategy will be successful, as indeed, it ultimately is not.)

This completes the proof.

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