Mathematics: The Loss of Certainty. by Morris Kline. Oxford. 366 pp. $19.95.

Professor Kline recounts a series of "shocks", "disasters" and "shattering" experiences leading to a "loss of certainty" in mathematics. However, he doesn't mean that the astronaut should mistrust the computations that tell him that firing the rocket in the prescribed direction for the prescribed number of seconds will get him to the moon.

The ancient Greeks were "shocked" to discover that the side and diagonal of a square could not be integer multiples of a common length. This spoiled their plan to found all mathematics on that of whole numbers. Nineteenth century mathematics was "shattered" by the discovery of non-Euclidean geometry (violating Euclid's axiom that there is exactly one parallel to a line through an external point), which showed that Euclidean geometry isn't based on self-evident axioms about physical space (as most people believed). Nor is it a necessary way of thinking about the world (as Kant had said).

Once detached from physics, mathematics developed on the basis of the theory of sets, at first informal and then increasingly axiomatized, culminating in formalisms so well described that proofs can be checked by computer. However, Gottlob Frege's plausible axioms led to Bertrand Russell's surprising paradox of the the set of all sets that are not members of themselves. (Is it a member of itself?). L.E.J. Brouwer reacted with a doctrine that only constructive mathematical objects should be allowed (making for a picky and ugly mathematics), whereas David Hilbert proposed to prove mathematics consistent by showing that starting from the axioms and following the rules could never lead to contradiction. In 1931 Kurt Goedel showed that Hilbert's program cannot be carried out, and this was another surprise.

However, Hilbert's program and Tarski's work led to metamathematics, which studies mathematical theories as mathematical objects. This replaced many of the disputes about the foundations of mathematics by the peaceful study of the structure of the different approaches.

Professor Kline's presentation of these and other surprises as shocks that made mathematicians lose confidence in the certainty and in the future of mathematics seems overdrawn. While the consistency of even arithmetic cannot be proved, most mathematicians seem to believe (with Goedel) that mathematical truth exists and that present mathematics is true. No mathematician expects an inconsistency to be found in set theory, and our confidence in this is greater than our confidence in any part of physics.