Mid Term for CS222 2005

Tom Costello

Computer Science Department, Stanford University, Stanford, CA 94305,

Email costello@cs.Stanford.EDU

Please submit your answers by Midnight Wednesday February 16th to the email address above.

We extend First Order Logic with a new term forming operator. It takes three arguments, a first order formula, and two terms. It can be read, $\phi ? t_1 : t_2$ if ϕ then t_1 else t_2. It can be defined as

$$\psi(\phi ? t_1 : t_2) \equiv \phi \supset \psi(t_1) \wedge \neg\phi \supset \psi(t_2) \tag{1}$$

We add the following rules.

$$\frac{\Gamma, \phi \Rightarrow \psi(t_1), \Delta \qquad \Gamma \Rightarrow \psi(t_2), \phi, \Delta}{\Gamma \Rightarrow \psi(\phi ? t_1 : t_2), \Delta}$$

$$\frac{\Gamma, \psi(t_2) \Rightarrow \phi, \Delta \qquad \Gamma, \phi, \psi(t_1) \Rightarrow \Delta}{\Gamma, \psi(\phi ? t_1 : t_2) \Rightarrow \Delta}$$

1. Derive the definition using the rules. ($\phi \equiv \psi$ is a shorthand for $\phi \supset \psi \wedge \psi \supset \phi$).
2. Write definitions for terms, atomic formula, and formulas for the new language.
3. Write a definition for free variables of a formula.
4. Write the truth definition for conditional expressions. I.e. define $\mathfrak{M} \models P(t_1, \ldots, t_n, (\phi ? t'_1 : t'_2), t_n + 1, \ldots, t_m)$.
5. Show the rules are sound.

6. Explain why the construction of Henkin theories remains the same, and why the proof that T^* is conservative over T remains valid.

7. Explain why the definion of T_m is unchanged, and why the lemma "If T_m is a maximally consistent extension of T_ω then T_m is a Henkin theory" is still valid despite the changes in the definitions of formulas and terms.

8. Modify the construction of the model $\mathfrak{A}$ of T_m to take care of the new conditional expressions.

9. Show that $T_m \vdash \psi(\phi?t_1\colon t_2)$ iff $\mathfrak{A} \models \psi(\phi?t_1\colon t_2)$.

10. Where in your proof did you use each rule?